134. *On Baire Functions on Infinite Product Spaces.*

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A set will be called a Baire set if its characteristic function is a Baire function. In euclidean space, it is well known that the set of all Baire sets coincides with the set of all borel sets. But in general this is not true for other spaces. Of course in any space, it is evident that the set of Baire sets is contained in the set of borel sets. But the converse is not true, namely the characteristic function of a Borel set is not always a Baire function. Such an example is easily derived in an infinite product space, by using some property of Baire functions on this space.

Theorem 1. The value of real valued continuous function on the product space of closed intervals [0 1] is determined by at most countable coordinates. Namely let $\Omega=\prod_{\alpha\in\Theta}\Omega_\alpha$, where for any α $\Omega_\alpha=[0\ 1]$ and Θ is a set of indexes. For any continuous function $f(p)$ on Ω, there exist at most countable coordinates $\alpha_1, \alpha_2, \dots$ depending on $f(p)$, such that for any two points $p=\prod_{\alpha\in\Theta}p_\alpha$, $q=\prod_{\alpha\in\Theta}q_\alpha$ of Ω $f(p)=f(q)$ when $p_{\alpha_i}=q_{\alpha_i}$ $(i=1, 2, \dots)$

Proof. We define the continuous function $f_\alpha(p)$ by $f_\alpha(p)=p_\alpha$. Let R be the smallest Ring of real-valued continuous functions which contains all $f_\alpha(p)$. Then for any two different points q and r there exists $f_\alpha(p)$ such that $f_\alpha(q)\neq f_\alpha(r)$. By a theorem of Gelfand-Silov[1] we see that any continuous function $f(p)$ on Ω may uniformly be approximated by a sequence of elements of R. On the other hand the element of R is the function which depends only on finite coordinates. So $f(p)$ is a function which depends on at most countable coordinates.

Theorem 2. The value of continuous function on the product space of bicompact spaces is determined by at most countable coordinates.

Proof. Let $\Omega=\prod_{\alpha\in\Theta}\Omega_\alpha$, where for any α Ω_α is a bicompact space and Θ is a set of indexes. By the well known theorem every bicompact space may be embedded homeomorphically in an infinite product of intervals [0 1]. So every Ω_α can be embedded homeomorphically in $\bar{\Omega}_\alpha=\prod_{\beta\in\Theta_\alpha}\Omega_\beta$, where for any β $\Omega_\beta=[0\ 1]$ and Θ_α is a set of indexes. We put $\bar{\Theta}=\bigcup_\alpha\Theta_\alpha$ and let $\bar{\Omega}=\prod_{\alpha\in\Theta}\bar{\Omega}_\alpha=\prod_{\beta\in\bar{\Theta}}\Omega_\beta$. Then Ω can be embedded homeomorphically in $\bar{\Omega}$. Since Ω is bicompact, the homeomorphic image Ω' of Ω is closed in $\bar{\Omega}$. So every continuous function on Ω' can always be extended to a continuous function on $\bar{\Omega}$. In virtue of Theorem 1 any continuous function $f(p)$ on $\bar{\Omega}$ is determined by at most countable

1) Rec. Math, **9.7** (1941), 25.