On the generalized Nörlund summability of a sequence of Fourier coefficients

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1. Introduction. Let $f(t)$ be a periodic function with period 2π on $(-\infty, \infty)$ and Lebesgue integrable over $(-\pi, \pi)$. Then the conjugate series of the Fourier series

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt)$$

of f is

$$\sum_{n=1}^{\infty}(b_n \cos nt - a_n \sin nt) \equiv \sum_{n=1}^{\infty} B_n(t).$$

Since Fejer [3] found the relations between the "jump" of $f(t)$ at $t = x$ and the sequence $\{nB_n(x)\}$, there are many results which show how the behaviour of $f(t)$ in the neighborhood of $t = x$ controls the convergence of the sequence $\{nB_n(x)\}$ to the jump in the sense of summability. To state the most recent result of Khare and Tripathi [5], we need the following definitions.

Given two sequences $p = \{p_n\}$ and $q = \{q_n\}$, the convolution $(p * q)$ is defined by

$$(p * q)_n = \sum_{k=0}^{n} p_{n-k} q_k = \sum_{k=0}^{n} p_k q_{n-k}.$$

Let $\{s_n\}$ be a sequence. When $(p * q)_n \neq 0$ for all n, the generalized Nörlund transform of the sequence $\{s_n\}$ is the sequence $\{t_n^{p,q}\}$ obtained by putting

$$t_n^{p,q} = \frac{1}{(p * q)_n} \sum_{k=0}^{n} p_{n-k} q_k s_k.$$

If $\lim_{n\to\infty} t_n^{p,q}$ exists and is equal to s, then the sequence $\{s_n\}$ is said to be summable (N, p_n, q_n) to the value s.

If $s_n \to s$ $(n \to \infty)$ induces $t_n^{p,q} \to s$ $(n \to \infty)$, then the method (N, p_n, q_n) is called to be regular. The necessary and sufficient condition for (N, p_n, q_n) method to be regular is $\sum_{k=0}^{n} |p_{n-k}\ q_k| = O(|(p * q)_n|)$ and $p_{n-k} = o(|(p * q)_n|)$ as $n \to \infty$ for every fixed $k \geq 0$ (see Borwein [2]).

The method (N, p_n, q_n) reduces to the Nörlund method (N, p_n) if $q_n = 1$ for all n and to the Riesz method $(\overline{N}, q_n)$ if $p_n = 1$ for all n. We know that (N, p_n) mean or $(\overline{N}, q_n)$ mean includes as a special case Cesàro and harmonic means or logarithmic mean, respectively.

The method $(N, p_n, q_n)(C, 1)$ is obtained by superimposing the method (N, p_n, q_n) on the Cesàro mean (C,1) of order one (see Astrachan [1]).

Throughout this paper, we shall use the following notations:

$$\psi_x(t) = \{f(x + t) + f(x - t) - l\},$$
$$\Psi_x(t) = \int_0^t |\psi_x(u)| du,$$

for any fixed x $(-\infty < x < \infty)$ and a constant l depending on x. For two sequence $\{p_n\}$ and $\{q_n\}$, we define $P(t)$ $(0 \leq t < \infty)$ and R_n $(n = 0, 1, 2, \ldots)$ by

$$P(t) = \sum_{k=0}^{[t]} p_k \quad \text{and} \quad R_n = (p * q)_n = \sum_{k=0}^{n} p_{n-k} q_k,$$

where $[t]$ denotes the integral part of t.

Theorem KT (Khare and Tripathi [5]). *Let (N, p_n, q_n) be regular Nörlund method defined by a non-negative, non-increasing sequence $\{p_n\}$ and a non-negative, non-decreasing sequence $\{q_n\}$. If the condition*

$$\text{(1.1)} \quad \int_{\pi/n}^{\delta} \frac{|\psi_x(t)|}{t} P\left(\frac{\pi}{t}\right) dt = o(R_n q_n^{-1}) \quad (n \to \infty)$$

holds for a number δ, $0 < \delta < \pi$, then the sequence $\{nB_n(x)\}$ is summable $(N, p_n, q_n)(C, 1)$ to l/π.

In this paper, by generalizing a result of Hirokawa and Kayashima [4], we shall give a theorem which contains Theorem KT.

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