The Bergman kernel on weakly pseudoconvex tube domains in $\mathbf{C}^2$

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(Communicated by Heisuke HIRONAKA, M.J.A., Feb. 12, 1999)

Key words: Bergman kernel; Szegö kernel; weakly pseudoconvex; of finite type; tube; asymptotic expansion; real blowing-up.

1. Introduction. In this note, we announce a result in [14] of an asymptotic expansion of the Bergman kernel for certain class of weakly pseudoconvex tube domains of finite type in $\mathbf{C}^2$. We also give an analogous result of the Szegö kernel for the same class of tube domains.

Let Ω be a domain with smooth boundary in $\mathbf{C}^n$. The Bergman space $B(\Omega)$ is the subspace of $L^2(\Omega)$ consisting of holomorphic L^2-functions on Ω. The Bergman projection is the orthogonal projection $\mathbf{B} : L^2(\Omega) \to B(\Omega)$. It is known that the projection $\mathbf{B}$ can be represented by using some integral kernel:

$$\mathbf{B}f(z) = \int_\Omega K(z,w)f(w)dV(w) \quad \text{for } f \in L^2(\Omega).$$

where $K : \Omega \times \Omega \to \mathbf{C}$ is the *Bergman kernel* of the domain Ω and dV is the Lebesgue measure on Ω. In this paper we restrict the Bergman kernel on the diagonal of the domain and study the boundary behavior of $K(z) = K(z,z)$.

There are many studies about the boundary behavior of the Bergman kernel. First we consider the Bergman kernel $K(z)$ of a bounded strictly pseudoconvex domain Ω. Hörmander [12] showed that the limit of $K(z)d(z-z^0)^{n+1}$ at $z^0 \in \partial\Omega$ equals the determinant of the Levi form at z^0 times $n!/4\pi^n$, where d is the Euclidean distance. Diederich [5], [6] obtained analogous results for the first and mixed second derivatives of $K(z)$. Moreover C. Fefferman [8] and Boutet de Monvel and Sjöstrand [3] gave a very strong result of the boundary behavior of Bergman kernel. The Bergman kernel of Ω can be expressed in the following:

$$K(z) = \frac{\varphi(z)}{r(z)^{n+1}} + \psi(z)\log r(z), \tag{1.1}$$

where $r \in C^\infty(\overline{\Omega})$ is a defining function of Ω (i.e. $\Omega = \{r > 0\}$ and $|dr| > 0$ on $\partial\Omega$) and φ, $\psi \in C^\infty(\overline{\Omega})$ can be expanded asymptotically with respect to r.

On the other hand, there are not so strong results in the weakly pseudoconvex case. Let us recall important studies in this case. Many estimates of the size of the Bergman kernel have been obtained (see the references in [1]). In particular Catlin [4] gave a complete estimate from above and below for domains of finite type in $\mathbf{C}^2$. Recently Boas, Straube and Yu [1] and Diederich and Herbort [7] computed a boundary limit in the sense of Hörmander for a large class of domains of finite type in $\mathbf{C}^n$ on a nontangential cone. Though the above studies about *estimate* and *boundary limit* are detailed, clear asymptotic formula like (1.1) is yet to be obtained. In this note we give an asymptotic expansion of the Bergman kernel for certain class of weakly pseudoconvex tube domains of finite type in $\mathbf{C}^2$. Gebelt [9] and Haslinger [11] recently computed certain asymptotic formulas for the special cases, but the method of our expansion is different from theirs.

Our main idea used to analyse the Bergman kernel is to introduce certain real blowing-up. Since the set of strictly pseudoconvex points are dense on the boundary of the domain of finite type, it is a serious problem to resolve the difficulty caused by the influence of strictly pseudoconvex points near z^0. This problem can be avoided by restricting the argument on a non-tangential cone in the domain. We surmount the difficulty in the case of certain class of tube domains by blowing up at the weakly pseudoconvex point z^0 and introducing two new variables. The Bergman kernel can be developed asymptotically in terms of these variables.

Our method of the computation is based on the studies [8], [3], [2], [20]. Our starting point is certain integral representation of the Bergman kernel in [17],

1991 Mathematics Subject Classification. 32A40, 32F15, 32H10.