K-approximations and strongly countable-dimensional spaces

By Yasunao HATTORI

Department of Mathematics, Shimane University, 1060 Nishi-Kawatsu-cho, Matsue, Shimane 690-8504

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1. Introduction. Throughout the present paper, by the dimension we mean the covering dimension dim. We shall consider a characterization of a class of infinite dimensional metrizable spaces in terms of K-approximations. In [5], Dydak-Mishra-Shukla introduced a concept of a K-approximation of a mapping to a metric simplicial complex and characterized n-dimensional spaces and finitistic spaces in terms of K-approximations. Let X be a space, K a metric simplicial complex and $f : X \to K$ a continuous mapping. A mapping $g : X \to K$ is said to be a *K-approximation of f* if for each simplex $\sigma \in K$ and each $x \in X$, $f(x) \in \sigma$ implies $g(x) \in \sigma$. A K-approximation $g : X \to K$ of f is called an *n-dimensional K-approximation* if $g(X) \subset K^{(n)}$ and a *finite dimensional K-approximation* if $g(X) \subset K^{(m)}$ for some natural number m, where $K^{(m)}$ denotes the m-skelton of K.

The concept of finitistic spaces was introduced by Swan [12] for working in fixed point theory and is applied to the theory of transformation groups by using the cohomological structures (cf. [1]). For a family $\mathcal{U}$ of a space X the order $\operatorname{ord}\mathcal{U}$ of $\mathcal{U}$ is defined as follows: $\operatorname{ord}_x\mathcal{U} = |\{U \in \mathcal{U} : x \in U\}|$ for $x \in X$ and $\operatorname{ord}\mathcal{U} = \sup\{\operatorname{ord}_x\mathcal{U} : x \in X\}$. We say a family $\mathcal{U}$ has *finite order* if $\operatorname{ord}\mathcal{U} = n$ for some natural number n. A space X is said to be *finitistic* if every open cover of X has an open refinement with finite order. We notice that finitistic spaces are also called *boundedly metacompact spaces* (cf. [7]). It is obvious that all compact spaces and all finite dimensional paracompact spaces are finitistic spaces. More precisely, we have a useful characterization of finitistic spaces.

Proposition ([5], [8]). *A paracompact space X is finitistic if and only if there is a compact subspace C of X such that $\dim F < \infty$ for every closed subspace F with $F \cap C = \emptyset$.*

The dimension-theoretic properties of finitistic spaces are investigated by several authors (cf. [3], [4], [5] and [8]). In particular, Dydak-Mishra-Shukla ([5]) proved the following.

Theorem A ([5]). *For a paracompact space X the following are equivalent.*

(a) $\dim X \leq n$.

(b) *For every metric simplicial complex K and every continuous mapping $f : X \to K$ there is an n-dimensional K-approximation g of f.*

(c) *For every metric simplicial complex K and every continuous mapping $f : X \to K$ there is an n-dimensional K-approximation g of f such that $g|f^{-1}(K^{(n)}) = f|f^{-1}(K^{(n)})$.*

Theorem B ([5]). *For a paracompact space X the following are equivalent.*

(a) *X is a finitistic space.*

(b) *For every metric simplicial complex K and every continuous mapping $f : X \to K$ there is a finite dimensional K-approximation g of f.*

(c) *For every integer $m \geq -1$, every metric simplicial complex K and every continuous mapping $f : X \to K$ there is a finite dimensional K-approximation g of f such that $g|f^{-1}(K^{(m)}) = f|f^{-1}(K^{(m)})$.*

The purpose of the present note is to extend Theorem A to a class of metrizable spaces that have strong large transfinite dimension.

For a metric space (X, ρ), a subset A of X and $\varepsilon > 0$ we denote $S_\varepsilon(A) = \{x \in X : \rho(x, A) < \varepsilon\}$. We denote the set of natural numbers by ω. We refer the reader to [6] and [11] for basic results in dimension theory.

2. Results. We begin with the definition of strong small transfinite dimension introduced by Borst [2]. A normal space X is said to *have strong small transfinite dimension* if for every non-empty

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