Uniqueness of the nonlinear term of a boundary value problem from the first bifurcating branch

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1. Statement of results. In this paper we shall consider the inverse problem of determining the nonlinear term g of the boundary value problem

$$(1.1)\quad \begin{cases} u'' + [\lambda - q(x)]u = g(u), \ 0 < x < 1, \\ u'(0) = u(1) = 0, \end{cases}$$

from its first bifurcating branch. From a viewpoint of physical applications, the investigation of the inverse problem can be regarded as a study to determine unknown inhomogeneity of elastic materials such as springs or rubbers by searching a modulus of elasticity which matches given period of vibration for each amplitude. Related inverse problems have been studied by Denisov [2], Lorenzi [8], Denisov and Lorenzi [3], Kamimura [7], which can be considered as investigations to determine inhomogeneity by measuring the dependence of the initial velocity and the displacement at a fixed time on the modulus of elasticity. There have been few investigations concerning inverse problems of determining unknown nonlinear terms in nonlinear differential equations from some measured date for their solutions, outside of these works.

An existence result for the inverse problem mentioned in the beginning was established by Iwasaki and Kamimura [4]. The purpose of the present paper is to establish a uniqueness result for the problem.

Let q be a real function of class $C[0, 1]$ and assume that g is a real function of class $C^1(\boldsymbol{R})$ satisfying $g(0) = g'(0) = 0$. As a representation of the first bifurcating branch of (1.1) in $\boldsymbol{R}^2$, let $\Gamma(g)$ be the set of $(\lambda, h) \in \boldsymbol{R}^2$ for which there exists a solution $u(x)$ of (1.1) such that (i) $u(x) \neq 0$ for any $x \in [0, 1)$; (ii) $u(0) = h$. The assumption $g(0) = g'(0) = 0$ implies that the linearized problem of (1.1) at the trivial solution $u(x) \equiv 0$ is :

$$(1.2)\quad \begin{cases} u'' + [\lambda - q(x)]u = 0, \ 0 < x < 1, \\ u'(0) = u(1) = 0. \end{cases}$$

Therefore the set $\Gamma(g)$ bifurcates at the point $(\lambda_1, 0)$ from the trivial solution $u(x) \equiv 0$, where λ_1 is the first eigenvalue of the problem (1.2) (see [4,§2], also see [1,9,10] for general theory).

Throughout the paper, we assume that the first eigenfunction $v_1(x)$ of (1.2) satisfies the following three conditions:

(A1) $v_1''(0) < 0$.

(A2) $v_1'(x) < 0$ for $0 < x \leq 1$.

(A3) $v_1''(x)v_1(x) \leq 2v_1'(x)^2$ for $0 \leq x < 1$.

This is an assumption on q. It should be pointed-out that if $\max_{0 \leq x \leq 1} q(x) < \lambda_1$ then (A1)−(A3) hold. For other sufficient conditions for (A1)−(A3) the reader may refer to [4, Remark 4.8].

We use the following two function spaces. Let $0 < \alpha < 1/2$ and let X, Y be function spaces defined by

$$X := \Big\{ g(h) \in C^1(\boldsymbol{R}) \,|\, g(0) = g'(0) = 0, \ \sup_{h,k \in R, h \neq k} \frac{|(1 + |k|^\alpha)g'(k) - (1 + |h|^\alpha)g'(h)|}{|k - h|^\alpha} < \infty \Big\},$$

$$Y := \Big\{ \lambda(h) \in C(\boldsymbol{R}) \,|\, h\lambda'(h) \in C(\boldsymbol{R}), \ \lambda(0) = \lambda_1, \ \sup_{h \in R} |\lambda(h)| + \sup_{h,k \in R, h \neq k} \frac{||k|^{3/2}(1 + |k|^\alpha)\lambda'(k) - |h|^{3/2}(1 + |h|^\alpha)\lambda'(h)|}{|k - h|^{\alpha + 1/2}} < \infty \Big\}.$$

For $\lambda(h) \in Y$, let $u(h, x)$ denote the solution of the initial value problem

$$(1.3)\quad \begin{cases} u'' + [\lambda(h) - q(x)]u = g(u), \\ u(0) = h, \ u'(0) = 0. \end{cases}$$

Clearly if $(\lambda(h), h) \in \Gamma(g)$ then $u(h, 1) = 0$.

The main result of the present paper is stated as follows:

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