On Standard L-Functions for Unitary Groups*)

By Keiji TAKANO

Department of Mathematics, Osaka University

(Communicated by Shokichi IYANAGA, M. J. A., Jan. 13, 1997)

Introduction. Let $G_n = U_{n,n}(\boldsymbol{K}/k)$ be the quasi-split unitary group of $2n$-dimension with respect to a quadratic extension $\boldsymbol{K}/k$ of number fields. The basic identity of Rankin-Selberg integral established in [4], [5] interpolates the *standard* L-function of a cuspidal automorphic representation of $G_n(\boldsymbol{A})$. In [5], the two of main steps in the theory of Rankin-Selberg method were carried out, though the group was $G_n = Sp_n$, or $O_{n,n}$;

(1) the investigation of analytic properties of the global Rankin-Selberg integral and,

(2) the computations of unramified local integrals.

These two parts can be carried out entirely in the same way as [5] also for our group $G_n = U_{n,n}(\boldsymbol{K}/k)$, which we shall state in §1.

The main part of this paper is devoted to the study of local integrals including finite ramified and archimedean places. We shall extend the method of [5] to adapt to representations that cannot be embedded in principal series representations. We rewrite these integrals by the Godement-Jacquet zeta integrals and obtain the analytic continuations of them. Then it is seen that, at finite ramified places, they can be made constant for a suitable choice of a test function, which enables us to prove the finiteness of poles of the partial standard L-function by the usual procedure of the Rankin-Selberg method.

The author would like to thank Takao Watanabe for many helpful advices and encouragement.

Notation. Let k be a number field and k_v be the completion of k at a place v of k. Let $G_n = U_{n,n}(\boldsymbol{K}/k)$ be the quasi-split form of unitary group of $2n$-dimension defined with respect to a quadratic extension $\boldsymbol{K}/k$ of number fields. The Galois involution of $\boldsymbol{K}/k$ is denoted by $x \mapsto \bar{x}$. We realize the group of k-points of G_n as

$$G_n(k) = \{g \in GL_{2n}(\boldsymbol{K}) \mid gJ_n{}^t\bar{g} = J_n\},$$
$$\text{where } J_n = \begin{pmatrix} 0 & 1_n \\ -1_n & 0 \end{pmatrix}.$$

We often write simply as $G_n = G_n(k)$, if there is no fear of confusion. If a place v splits (resp. remains prime) in $\boldsymbol{K}$, $G_n(k_v)$ is isomorphic to $GL_{2n}(k_v)$ (resp. $U_{n,n}(\boldsymbol{K}_v/k_v)$ where $\boldsymbol{K}_v = \boldsymbol{K} \otimes_k k_v$ is a quadratic field extension of k_v). Let $K_{n,v}$ be the standard maximal compact subgroup of $G_n(k_v)$.

Let T_n (resp. A_n) be the maximal k-torus (resp. maximal k-split torus) given by

$$T_n = \{\mathrm{diag}(t_1, \dots, t_n, \bar{t}_1^{-1}, \dots, \bar{t}_n^{-1}) \mid t_i \in \boldsymbol{K}^\times\},$$
$$A_n = \{\mathrm{diag}(a_1, \dots, a_n, a_1^{-1}, \dots, a_n^{-1}) \mid a_i \in k^\times\}$$

and let χ_i be the k-rational character of A_n defined by $\chi_i(\mathrm{diag}(a_1, \dots, a_n, a_1^{-1}, \dots, a_n^{-1})) = a_i$ for $1 \leqslant i \leqslant n$. Then $\{\chi_1, \dots, \chi_n\}$ forms a $\boldsymbol{Z}$-basis of $X^*(A_n) = \mathrm{Hom}(A_n, \boldsymbol{G}_m)$. Let $B_n = T_n \ltimes N_n$ be the Borel subgroup of G_n, of which the unipotent radical N_n is the subgroup consists of elements of the form $\begin{pmatrix} u & x \\ 0 & {}^t\bar{u}^{-1} \end{pmatrix}$, where $u \in GL_n(\boldsymbol{K})$ is upper triangular with ones in diagonals, and $x \in \mathrm{Mat}_n(\boldsymbol{K})$ is such that $x = {}^t\bar{x}$. Let $\Phi_n = \Phi(G_n, A_n)$ be the relative root system of G_n with respect to A_n and let Φ_n^+ be the set of positive roots corresponding to B_n explicitly given by $\Phi_n^+ = \{2\chi_i (1 \leqslant i \leqslant n), \chi_i \pm \chi_j (1 \leqslant i < j \leqslant n)\}$. Denote by $W_n = W(G_n, A_n)$ the relative Weyl group of G_n. For each $\alpha \in \Phi$, let $N_{n,\alpha}$ be the root subgroup determined by α.

For each integer r with $1 \leqslant r \leqslant n$, let $P_n^{(r)} = M_n^{(r)} \ltimes U_n^{(r)}$ be the maximal parabolic k-subgroup of G_n given by

$$M_n^{(r)} = \left\{ \iota\left(x, \begin{pmatrix} A & B \\ C & D \end{pmatrix}\right) = \left(\begin{array}{cc|cc} x & & & \\ & A & & B \\ \hline & & {}^t\bar{x}^{-1} & \\ & C & & D \end{array} \right) \;\middle|\; \begin{array}{l} x \in GL_r(\boldsymbol{K}), \\ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in G_{n-r} \end{array} \right\}$$

*) Partly supported by Reseach Fellowship of the Japan Society for the promotion of Science for Young Scientists.