Remarks on the Periodic Solution of the Heat Convection Equation in a Perturbed Annulus Domain

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1. Introduction. We consider the heat convection equation in a time-dependent bounded domain $\Omega(t)$ of $\boldsymbol{R}^2$ which varies periodically with period T_p.

$$(1)\quad \begin{cases} u_t + (u\cdot\nabla)u = -(\nabla p)/\rho + \\ \qquad \{1-\alpha(\theta - T_0)\}g + \nu\Delta u & \text{in } \hat{\Omega}, \\ \qquad \operatorname{div} u = 0 & \text{in } \hat{\Omega}, \\ \theta_t + (u\cdot\nabla)\theta = \kappa\Delta\theta & \text{in } \hat{\Omega}, \end{cases}$$

$$(2)\quad u|_{\partial\Omega(t)} = \beta(x, t),\ \theta|_{\Gamma_0} = T_0 > 0,\ \theta|_{\Gamma(t)} = 0 \quad \text{for any } t \in (0, \infty),$$

$$(3)\quad u(\cdot, t+T_p) = u(\cdot, t),\ \theta(\cdot, t+T_p) = \theta(t), \quad \text{in } \Omega(0),$$

where $\hat{\Omega} = \cup_{0<t<\infty}\Omega(t)\times\{t\}$ and $\partial\Omega(t)$ (the boundary of $\Omega(t)$) consists of two smooth components, i.e. $\partial\Omega(t) = \Gamma_0 \cup \Gamma(t)$, and Γ_0 is the inner boundary which bounds a compact set K, while the outer boundary $\Gamma(t)$ is a smooth one with respect to both x and t. We assume that the set K includes the origine O and $\Omega(t)$ is included in a ball $B_d = B(O, d/2)$. We put $B = B_d\setminus K$. Moreover, $u = u(x, t)$ is the velocity vector, $p = p(x, t)$ is the pressure and $\theta = \theta(x, t)$ is the temperature; ν, κ, α, ρ and $g = g(x)$ are the kinematic viscosity, the thermal conductivity, the coefficient of volume expansion, the density at $\theta = T_0$ and the gravitational vector, respectively. (Hereafter, we denote the heat convection equation by HC equation).

As for the 3-dimensional problems, we proved the existence, uniqueness and the stability of the periodic strong solutions in [9] and [10] when the data are small, while Morimoto [8] obtained the periodic weak solutions. Recently, Inoue-Ôtani [6] studied and got the periodic strong solution under their small type conditions when the space dimention $n = 2$ or 3 (in time-dependent domains). On the other hand , for the 2-dimensional cases, we obtained, in [14], a sufficient condition for the existence of the periodic strong solution in the form of a certain relation between given data including a time period, but not including the magnitude of b which is an extension of the boundary function $\beta(x, t)$. The purpose of the present paper is to improve the result of our previous one [14] and to remove the small type condition on the boundary data of the fluid velocity. (We announced the results of this paper in [15]).

2. Preliminaries. First, we make assumptions:

(A1) For each fixed $t \geq 0$, $\Gamma(t)$ and Γ_0 are both simple closed curves. Moreover, they are smooth (of class C^∞) in x, t.

(A2) There exists $\Omega(r_0, r_1) = \{x \in \mathbf{R}^2; 0 < r_0 < |x| < r_1\}$ such that $\Omega(r_0, r_1) \subset \Omega(t)$ for all $t \geq 0$. Moreover, there is $\delta > 0$ such that

$$\operatorname{dist}(\Gamma_0, \{|x| = r_0\}) \geq \delta \text{ and}$$
$$\operatorname{dist}(\Gamma(t), \{|x| = r_1\}) \geq \delta \text{ for all } t \geq 0.$$

(A3) $(t + T_p) = \Omega(t)$, $\Gamma(t+T_p) = \Gamma(t)$ and $\beta(\cdot, t+T_p) = \beta(\cdot, t)$ for all $t \geq 0$.

(A4) $g(x)$ is a bounded continuous vector function in $\boldsymbol{R}^2\setminus K$.

(A5) There exists a function $b = b(x, t)$ of the form $b = \operatorname{rot} c(x, t)$ where $c = (x, t) \in C^3$ on $B\times[0, \infty)$, periodic in t with period T_p and $b|_{\partial\Omega(t)} = \beta$.

Remark 1. By (A5), retaking $c(x, t)$, if necessary, it holds

$$\int_{\Gamma_0}\beta\cdot n dS = \int_{\Gamma(t)}\beta\cdot n dS = 0, \text{ where } B \text{ stands for } B_d\setminus K.$$

Here, we state two lemmas.

Lemma 2.1 (cf. Temam [19]). *For an arbitrary $\varepsilon > 0$, there exists $b_\varepsilon = b_\varepsilon(x, t)$ such that*

$$b_\varepsilon \in H^2(B),\ \operatorname{div} b_\varepsilon = 0,\ b_\varepsilon(\partial\Omega(t)) = \beta,$$
$$|((u\cdot\nabla)b_\varepsilon, u)| \leq \varepsilon\|\nabla u\|^2 \text{ for } u \in H^1_\sigma(\Omega(t)).$$

Lemma 2.2 ([12]). *For each $\varepsilon > 0$, there exists $\bar{\theta}_\varepsilon = \bar{\theta}_\varepsilon(x, t)$ such that $\bar{\theta}_\varepsilon \in C(\bar{B}) \cap H^2(B)$, $\bar{\theta}_\varepsilon(\Gamma_0) = T_0$, $\bar{\theta}_\varepsilon(\Gamma(t)) = 0$ and $\|(u\cdot\nabla)\bar{\theta}_\varepsilon\| \leq \varepsilon\|\nabla u\|$ for $u \in H^1_0(\Omega(t))$.*

Remark 2. $H^k(B)$ and $H^k_0(B)$ stand for Sobolev spaces. $H_\sigma(B)$ and $H^1_\sigma(B)$ mean sole-