On the Local Energy Decay of Higher Derivatives of Solutions for the Equations of Motion of Compressible Viscous and Heat-conductive Gases in an Exterior Domain in $\mathbf{R}^3$

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1. Introduction. Let Ω be an exterior domain in $\mathbf{R}^3$ with compact smooth boundary $\partial\Omega$. We consider the following system

$$(1.1)\quad \begin{cases} \rho_t + \gamma \operatorname{div} v = 0 & \text{in } [0,\infty)\times\Omega, \\ v_t - \alpha\Delta v - \beta\nabla(\operatorname{div} v) + \gamma\nabla\rho \\ \qquad\qquad + \omega\nabla\theta = 0 & \text{in } [0,\infty)\times\Omega, \\ \theta_t - \kappa\Delta\theta + \omega\operatorname{div} v = 0 & \text{in } [0,\infty)\times\Omega, \\ v|_{\partial\Omega} = 0,\ \theta|_{\partial\Omega} = 0 & \text{on } [0,\infty)\times\partial\Omega \\ (\rho, v, \theta)(0,x) = (\rho_0, v_0, \theta_0)(x) \\ \qquad\qquad & \text{in } \Omega, \end{cases}$$

where ρ is the density, $v = {}^T(v_1, v_2, v_3)$ the velocity and θ the absolute temperature, α, γ, κ, and ω are positive numbers and β is a non-negative number. This system is the linearized equation of motion of compressible viscous and heat-conductive gases in an exterior domain in $\mathbf{R}^3$, which was given by Matsumura and Nishida [6] and Ponce [9]. Concerning the nonlinear problem, the unique existence of smooth solutions globally in time near constant state $(\bar\rho_0, 0, \bar\theta_0)$ was studied by Matsumura and Nishida [8]. Deckelnick [2,3] proved the decay estimates for the solutions of nonlinear problem although the decay rate is weaker than that of Cauchy problem given by Matsumura and Nishida [6,7] and Ponce [9]. Our purpose is to get the decay estimates corresponding to Cauchy problem in the case of an exterior domain, which will be discussed in the forthcoming paper [5]. In our strategy, 1st step is to get local energy decay for the solutions of linearized equations (1.1). Kobayashi [4] proved the local energy decay of lower order derivatives of solutions. But since this system (1.1) is hyperbolic-parabolic type and since the regularity of solutions seems to be governed by the hyperbolic part ρ, we shall need to prove the regularity of solutions. Therefore in this paper we discuss a local energy decay estimates for higher order derivatives of solutions for the linearized equations.

Now we shall state the main results. Let $1 < q < \infty$, m be an integer and set

$$\boldsymbol{X}_q^m(\Omega) = \{{}^T\boldsymbol{U} : \boldsymbol{U} \in W_q^{m+1}(\Omega) \times \mathbf{W}_q^m(\Omega) \times W_q^m(\Omega)\},\ \boldsymbol{X}_q(\Omega) = \boldsymbol{X}_q^0(\Omega)$$

where ${}^T\boldsymbol{U}$ means the transposed $\boldsymbol{U}$, $W_q^m(\Omega) = \{u \in L_q(\Omega) : \|u\|_{m,q,\Omega} = (\sum_{|\alpha|\le m}\int_\Omega |\partial_x^\alpha u|^q dx)^{1/q} < \infty\}$ denotes the usual Sobolev spaces and $\mathbf{W}_q^m(\Omega) = \{W_q^m(\Omega)\}^3$. Define the 5×5 matrix operator $\boldsymbol{A}$ by the relation:

$$\boldsymbol{A} = \begin{pmatrix} 0 & \gamma\operatorname{div} & 0 \\ \gamma\nabla & -\alpha\Delta - \beta\nabla\operatorname{div} & \omega\nabla \\ 0 & \omega\operatorname{div} & -\kappa\Delta \end{pmatrix}$$

with the domain:

$$\mathcal{D}(\boldsymbol{A}) = \{{}^T\boldsymbol{U} = (\rho, v, \theta) \in W_q^1(\Omega)\times\mathbf{W}_q^2(\Omega) \times W_q^2(\Omega) : v|_{\partial\Omega} = 0,\ \theta|_{\partial\Omega} = 0 \text{ on } \partial\Omega\}.$$

Let $\boldsymbol{P}$ be the projection from $\mathcal{D}(\boldsymbol{A})$ into $\{{}^T(v,\theta) \in \mathbf{W}_q^2(\Omega)\times W_q^2(\Omega);\ v|_{\partial\Omega} = 0,\ \theta|_{\partial\Omega} = 0 \text{ on } \partial\Omega\}$ Then by Kobayashi [4], $-\boldsymbol{A}$ is a closed linear operator in $\boldsymbol{X}_q(\Omega)$ and the resolvent set contain $\sum = \{\lambda \in \mathbf{C} : C\mathrm{Re}\lambda + (\mathrm{Im}\lambda)^2 > 0\}$ where C is a constant depending only on α, β, γ, κ, and ω. Moreover, the following properties are valid; There exist positive constants λ_0 and $\delta < \frac{\pi}{2}$ such that

$$(1.2)\quad |\lambda|\,\|(\lambda+\boldsymbol{A})^{-1}\boldsymbol{F}\|_{\mathrm{X}_q(\Omega)} + \|\boldsymbol{P}(\lambda+\boldsymbol{A})^{-1}\boldsymbol{F}\|_{2,q,\Omega} \le C(\lambda_0, \delta, m)\,\|\boldsymbol{F}\|_{\mathrm{X}_q(\Omega)}$$

for any $\lambda - \lambda_0 \in \sum_\delta = \{\lambda \in \boldsymbol{C};\ |arg\lambda| \le \pi - \delta\}$ and any $\boldsymbol{F} \in \boldsymbol{X}_q(\Omega)$. This estimates means that $-\boldsymbol{A}$ generates an analytic semigroup e^{-tA} on $\boldsymbol{X}_q(\Omega)$.

Let b be a positive number such that $\partial\Omega \subset B_b = \{x \in \mathbf{R}^3 : |x| < b\}$. Set

$$\boldsymbol{Y}_{q,b}^m(\Omega) = \{\boldsymbol{U} = {}^T(\rho, v, \theta) \in \boldsymbol{X}_q^m(\Omega) : \boldsymbol{U}(x) = 0 \text{ for } x \in \mathbf{R}^3\backslash B_b,\ \int_{\Omega_b}\rho(x)\,dx = 0\},$$

and $\boldsymbol{Y}_{q,b}(\Omega) = \boldsymbol{Y}_{q,b}^0(\Omega)$ where $\Omega_b = B_b \cap \Omega$. Then