Table of Quotient Curves of Modular Curves $X_0(N)$ with Genus 2

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1. Introduction. Let $X_0(N)$ be the modular curve corresponding to the modular group $\Gamma_0(N)$. All hyperelliptic curves $X_0(N)$ have been determined by Ogg [8], and their defining equations are given in [10]. Let $W(N)$ be the subgroup of $\mathrm{Aut}X_0(N)$ generated by all Atkin-Lehner's involutions ([1]), and let $W'(N)$ be a subgroup of $W(N)$. Then, by [2], we see that there are 138 cases in the range $N \leq 300$ such that the quotient curve $X' = X_0(N)/W'(N)$ is of genus two. For $N \geq 301$, we can find that there are only four such cases, see [4]. In this article, we shall give the table of defining equations of such X''s. We use the following

Theorem. *Let $S_2(N)$ be the space of cuspforms of weight two on $\Gamma_0(N)$ and $S_2(N)^{W'(N)}$ be the fixed part of $S_2(N)$ by $W'(N)$. Assume that $X' = X_0(N)/W'(N)$ is of genus two, and $S_2(N)^{W'(N)}$ is spanned by*

$$f_1 = \sum_{n\geq 1} a_n q^n,\ f_2 = \sum_{n\geq 2} b_n q^n$$

with $a_n, b_n \in \boldsymbol{Z}$, $a_1 b_2 \neq 0$. Put $z = f_1/f_2$ and $w = (f_2/q)^{-1}(dz/dq)$. Then z and w generate the function field $\boldsymbol{Q}(X')$ of X' over $\boldsymbol{Q}$ and give the defining equation $w^2 = f(z)$ of X'.

Proof. See [7].

f_1 and f_2 are computed by using Brandt matrices and trace formulas of Hecke operators ([5], [9]).

2. Results. The following is the list of $X_0(N)/W'(N)$ with genus equal to 2. The fourth column gives $f(z)$ in the theorem. The sixth column gives the dimension of the subspace of $S_2(N)^{W'(N)}$ spanned by newforms, and the seventh column gives the field of Hecke eigenvalues if $S_2(N)^{W'(N)}$ is $\boldsymbol{Q}$-simple and spanned by newforms.

N	$p\mid N$	$W'(N)$	$f(z)$	$\mathrm{disc}(f)$		
22	2,11	1	$(z^3+2z^2-4z+8)(z^3-2z^2+4z-4)$	$-2^{24}11^4$	0	
23	23	1	$(z^3-z+1)(z^3-8z^2+3z-7)$	$-2^{12}23^6$	2	$\boldsymbol{Q}(\sqrt{5})$
26	2,13	1	$z^6-8z^5+8z^4-18z^3+8z^2-8z+1$	$-2^{20}13^3$	2	
28	2,7	1	$(z^2+7)(z^2-z+2)(z^2+z+2)$	$2^{28}7^3$	0	
29	29	1	$z^6-4z^5-12z^4+2z^3+8z^2+8z-7$	$-2^{12}29^5$	2	$\boldsymbol{Q}(\sqrt{2})$
30	2,3,5	$\langle W_2\rangle$	$(z^2-z+1)(z^2+3z+1)(z^2-6z+1)$	$2^{13}3^9 5^5$	1	
		$\langle W_3\rangle$	$z(z+3)(z^2-z+4)(z^2-2z+5)$	$-2^{24}3^3 5^5$	0	
		$\langle W_{10}\rangle$	$(z^2-z-1)(z^2+2z-7)(z^2+3z-9)$	$-2^{13}3^6 5^2$	1	
31	31	1	$(z^3-2z^2-z+3)(z^3-6z^2-5z-1)$	$-2^{12}31^4$	2	$\boldsymbol{Q}(\sqrt{5})$
33	3,11	$\langle W_3\rangle$	$(z-1)(z^2-4z-8)(z^3+z^2+3z-1)$	$2^{12}3\cdot 11^9$	1	
35	5,7	$\langle W_7\rangle$	$(z+1)(z^2+4)(z^3-5z^2+3z-19)$	$-2^{12}5^7 7^3$	2	$\boldsymbol{Q}(\sqrt{17})$
37	37	1	$z^6+8z^5-20z^4+28z^3-24z^2+12z-4$	$-2^{12}37^3$	2	
38	2,19	$\langle W_2\rangle$	$(z^3-5z^2-4)(z^3+z^2-z+3)$	$-2^{15}19^6$	1	
39	3,13	$\langle W_{13}\rangle$	$(z-1)(z+3)(z^2-5z+3)(z^2+3z-1)$	$-2^{12}3^{10}13^2$	2	$\boldsymbol{Q}(\sqrt{2})$
40	2,5	$\langle W_8\rangle$	$(z^2+4z-4)(z^4+4z^2-8z+4)$	$-2^{25}5^5$	1	
		$\langle W_5\rangle$	$(z^2+4)(z^4+12z^2+16)$	$2^{36}5^2$	0	
42	2,3,7	$\langle W_3\rangle$	$(z^2-5z+1)(z^4+z^3+4z^2+z+1)$	$-2^{25}3\cdot 7^3$	1	
		$\langle W_6\rangle$	$(z^2+z-5)(z^4-z^3-2z^2-5z+11)$	$-2^{15}3\cdot 7^3$	0	
		$\langle W_{21}\rangle$	$(z^2+z+1)(z^4+7z^3+16z^2+7z+1)$	$2^{19}3\cdot 7^2$	1	
		$\langle W_{42}\rangle$	$(z^2-5z+7)(z^4-7z^3+22z^2-35z+23)$	$2^{13}3\cdot 7^2$	0	