Accessibility of Infinite Dimensional Brownian Motion to Holomorphically Exceptional Set*)

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1. Introduction. In [6], we introduced the notion of *holomorphically exceptional sets* of the complex Wiener space. In particular, we pointed out the following remarkable relation between holomorphically exceptional sets and the standard Brownian motion $(Z_t)_{t\geq 0}$ on the complex Wiener space: *Z_t does not hit a holomorphically exceptional set until time* 1 *almost surely.*

In any finite dimensional space, if the Brownian motion does not hit a certain set until time 1 almost surely, neither does it after time 1. So one may guess that the infinite dimensional Brownian motion never hits a holomorphically exceptional set after time 1, either.

But we will show in the present paper that the above guess is false. That is, we will construct a holomorphically exceptional set which the Brownian motion $(Z_t)_{t\geq 0}$ hits after a certain time $t_0 > 1$ almost surely.

The reason why such an example can exist lies essentially in a fact that the distributions of $(Z_t)_{t\geq 0}$ at different times are mutually singular.

2. Presentation of Theorem. Let (B, H, μ) be a *real* abstract Wiener space, i.e., B is a real separable Banach space (whose dimension is infinite), H is a real separable Hilbert space continuously and densely imbedded in B and μ is a Gaussian measure satisfying

$$\int_B \exp(\sqrt{-1}\langle z, l\rangle)\mu(dz) = \exp\left(-\frac{1}{4}\| l \|_{H^*}^2\right) \quad l \in B^* \subset H^*.$$

We introduce an *almost complex structure* $J : B \to B$ which is an isometry such that $J^2 = -1$ and that the restriction $J|_H : H \to H$ is also an isometry. The abstract Wiener space (B, H, μ) endowed with the almost complex stucture J is called an *almost complex abstract Wiener space* and denoted by (B, H, μ, J).

Let B^{*C} be the complexification of the dual space B^*. Then define

$$B^{*(1,0)} := \{\varphi \in B^{*C} \mid J^*\varphi = \sqrt{-1}\varphi\},$$
$$B^{*(0,1)} := \{\varphi \in B^{*C} \mid J^*\varphi = -\sqrt{-1}\varphi\}.$$

In other words, $B^{*(1,0)}$ is the space of bounded *complex linear* functionals on B and $B^{*(0,1)}$ is the space of bounded *complex anti-linear* functionals on B. We see that $B^{*C} = B^{*(1,0)} \oplus B^{*(0,1)}$. The Hilbert spaces H^{*C}, $H^{*(1,0)}$ and $H^{*(0,1)}$ are similarly defined.

Definition. 1. A function $G : B \to \boldsymbol{C}$ is called a *holomorphic polynomial,* if it is expressed in the form

(1) $G(z) = g(\langle z, \varphi_1\rangle, \ldots, \langle z, \varphi_n\rangle),\ z \in B,$

where $n \in \boldsymbol{N}$, $g : \boldsymbol{C}^n \to \boldsymbol{C}$ is a polynomial with complex coefficients and $\varphi_1, \ldots, \varphi_n \in B^{*(1,0)}$ The class of all holomorphic polynomials is denoted by $\mathscr{P}_h$.

Definition. 2. Let $p \in (1, \infty)$. For a sequence $\{G_n\} \subset \mathscr{P}_h$ such that $\sum_n \| G_n \|_{L^p(\mu)} < \infty$, we define a subset $N^p(\{G_n\})$ of B by

(2) $N^p(\{G_n\}) := \{z \in B \mid \sum_n | G_n(z) | = \infty\}.$

A set $A \subset B$ is called an *L^p-holomorphically exceptional set,* if it is a subset of a set of the type $N^p(\{G_n\})$. We denote the class of all L^p-holomorphically exceptional sets by $\mathcal{N}_h^p$. If an assertion holds outside of an L^p-holomorphically exceptional set, we say that it holds "$a.e.(\mathcal{N}_h^p)$".

Let $(Z_t)_{t\geq 0}$ be a B-valued independent increment process defined on a probability space $(\Omega, \mathscr{F}, P)$ such that $Z_0 = 0$ and the distribution of $Z_t - Z_s$, $t > s$, is μ_{t-s}, where $\mu_r(\cdot) := \mu(\cdot/\sqrt{r})$. Then the process $(Z_t)_{t\geq 0}$ becomes a diffusion process on B and it is called a *B-valued Brownian motion* (see, for example, [3]).

In [6], it is known that $(Z_t)_{t\geq 0}$ does not hit any L^p-holomorphically exceptional set until time 1 almost surely.

Theorem. *There exists an L^2-holomorphically exceptional set $A \subset B$ such that*

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