The Degree Function for Cellular Dynamics

By Takao NAMIKI

Department of Mathematical Sciences, University of Tokyo

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Abstract: To study the orbit structures of cellular dynamics, one have to study the entropy decreasing factor maps over some sofic systems. It is difficult to analyze the structure of these factor maps because the inverse image of almost every point is uncountable. In this paper, the author proposes a function called *the degree function,* a generalization of the degree of factor maps in finite to one cases, which indicates the exponential rate of the number of the inverse image of a word. Using the degree function, we get upper bounds of decreasing in spatial entropies and some relations.

1. Introduction and the background. It is well known that the cellular automata contains various orbit behaviors although its definition is simple. Given

Lattice. L, often using D-dimensional lattice $\boldsymbol{Z}^D$.

Cellular state space. fix a finite set (alphabet) A, for example $\{0,1\}$

Configuration space. $X = A^L$, or a sofic system (see section 2)

Neighborhood. a finite set Λ of lattice containing the origin

Interaction. a map $f : A^\Lambda \mapsto A$ (called a local map)

its dynamics $\tau : X \to X$ is defined $(\tau x)_s = f(x_t : t \in \sigma_s \Lambda)$ for all $x \in X$ where σ_t is the translation from origin to $t \in L$.

The notion of cellular automata has been recognized as an important model of "self organization". In the late forties J. von Neumann introduced the "29 states self reproducing automata", which is the origin of this stream [4].

But the universe seems to be more attractive from the viewpoint of dynamical systems theory. In the early eighties S. Wolfram [5] developed his numerical research on one-dimensional cellular automata as the target of dynamical systems and statistical mechanics. However, not so many results are obtained from mathematical viewpoint, especially from ergodic theory. In the present report, we call cellular automata over one-dimensional lattice $\boldsymbol{Z}$ as *cellular dynamics.*

The cellular dynamics are continuous maps with shift commuting property. Those are *factor maps* over sofic systems. When a factor map is surjective on the configuration space, it is well known that the factor map is boundedly finite to one [2]. These cases are deeply studied from the viewpoint of the isomorphism problem between topological Markov shifts [3].

But the case of not surjective, the topological entropy decreases. In these cases, the factor maps are uncountably infinite to one [2]. It is very difficult to use the standard tools that we have already known on maps of intervals and so on. In the present paper we introduce the degree function which is the number of n-word's inverse images. Then there are some relations between the degree function and spatial entropies as shown in section 4. We announce the results and the proofs will be published elsewhere.

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2. Notations. Let A be a finite set and σ the *shift transformation* on $A^{\boldsymbol{N}} = \{x = (x_n)_{n \in \boldsymbol{N}} ; x_n \in A\}$, i.e., $(\sigma x)_n = x_{n+1}$ $(n \in \boldsymbol{N})$. The shift transformation on $A^{\boldsymbol{Z}}$ defined in a similar way. A pair (X, σ) consisting of a σ-invariant set X and the restriction of σ to X, denoted again by σ, is called a (one-sided or two-sided) shift. If A is endowed with a topology and X is compact, then it