23. *Algebraic Geometry of Center Curves in the Moduli Space of the Cubic Maps*

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0. Introduction. In our previous paper [6], we have defined the so-called center curves BC_p and CD_p, which are algebraic curves, for the real cubic maps. The attached figure 1 gives the graphs of these curves for $p = 1, 2, 3, 4$. Note that these graphs exist only in the first and third quadrants. The same holds also for other values $p = 5, 6, \cdots$.

In the present paper we consider the complex maps. For such a cubic map g, we have two normal forms; $x^3 - 3Ax \pm \sqrt{B}$, $A, B \in \mathbf{C}$. Therefore, the complex affine conjugacy class of g can be represented by (A, B). The moduli space, consisting of all affine conjugacy classes of cubic maps, can be identified with the coordinate space $\mathbf{C}^2 = \{(A, B)\}$. For the post-critically finite complex cubic maps, the **center curves** CD_p, BC_p can be defined in the same way as in [6]. In section 1, we show how the equations of these curves are obtained by induction on p.

We can embed $\mathbf{C}^2$ canonically in $\mathbf{P}^2(\mathbf{C}) : (A, B) \to (1 : A : B)$. Then an affine algebraic curve $V_0 = \{(A, B) \in \mathbf{C}^2 : h(A, B) = 0\}$ uniquely determines a projective algebraic curve $V = \{(C : A : B) \in \mathbf{P}^2(\mathbf{C}) : H(C : A : B) = 0\}$ in $\mathbf{P}^2(\mathbf{C})$ such that $h(A, B) = H(1 : A : B)$ and $V \cap \mathbf{C}^2 = V_0$.

Definition. For a center curve V_0, the corresponding projective algebraic curve V is called the **projective center curve**. We denote by PBC_p and PCD_p, these curves corresponding to BC_p and CD_p respectively.

In sections 2 and 3, we give some properties of these curves from the viewpoint of algebraic geometry ([1]).

1. The equations of center curves. Let $f(x) = x^3 - 3Ax + \sqrt{B}$, with critical points $\pm\sqrt{A}$.

The equation of curve BC1 is obtained as follows:

$$f(\sqrt{A}) - (-\sqrt{A}) = (-2A + 1)\sqrt{A} + \sqrt{B} = 0$$
$$f(-\sqrt{A}) - \sqrt{A} = (2A - 1)\sqrt{A} + \sqrt{B} = 0.$$

Therefore,

$$\mathrm{BC1} : B = A(2A - 1)^2.$$

The equation of curve CD1 is obtained as follows:

$$f(\sqrt{A}) - \sqrt{A} = (-2A - 1)\sqrt{A} + \sqrt{B} = 0,$$
$$f(-\sqrt{A}) - (-\sqrt{A}) = (2A + 1)\sqrt{A} + \sqrt{B} = 0.$$

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