19. *Complete Local (S_{n-1}) Rings of Type $n \geq 3$ are Cohen-Macaulay**)

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§1. Introduction. Let A be a local ring of dimension d with maximal ideal m. The *type* of A, denoted by $r(A)$, is defined to be the dimension of $\mathrm{Ext}_A^d(A/m, A)$ as a vector space over A/m. Then Gorenstein local rings are characterized as Cohen-Macaulay local rings of type one (Bass [1]). Vasconcelos [12, p.53] conjectured that the condition $r(A) = 1$ is sufficient to imply that A is Gorenstein (cf. [4, p. 30]). Foxby [4] proved this conjecture for local rings containing a field, for unmixed local rings and for local rings satisfying some other conditions (along with a conjecture for modules). The conjecture was proven in general by Roberts [9], using a minimal free resolution of a dualizing complex. By modifying Roberts' argument, Costa, Huneke and Miller [3] proved that complete local domains of type two are Cohen-Macaulay. They also showed that there exists a non-Cohen-Macaulay equidimensional complete local ring of type two and that there is a non-Cohen-Macaulay reduced complete local ring of type two. Improving their method, Marley [6] proved that unmixed local rings of type two are Cohen-Macaulay and asked if complete local (S_{n-1}) rings of type $n \geq 3$ are Cohen-Macaulay. Kawasaki [5] answered Marley's question in the affirmative for local rings containing a field, making use of Theorem 3 in Bruns [2]. In this note we show that the question has the affirmative answer in general, using Kawasaki's idea. We also give a generalization for modules corresponding to that in [5].

§2. Results. Let R be a commutative noetherian ring. For an R-module M and a prime ideal p, the i-th *Bass number* of M at p, denoted by $\mu^i(p, M)$, is defined to be $\lambda(\mathrm{Ext}_R^i(R/p, M)_p)$, where λ denotes length. Let $I^{\cdot}$ be a minimal injective resolution of M. Then $\mu^i(p, M)$ is equal to the number of copies of $E(R/p)$ which appear in I^i as a direct summand, where $E(R/p)$ denotes the injective envelope of R/p. For basic properties of Bass numbers, see Bass [1]. Let t be an integer. A finitely generated R-module M is said to be (S_t) if depth $M_p \geq \min\{t, \dim M_p\}$ for every p in $\mathrm{Supp}(M)$. In the following A always denotes a local ring of dimension d with maximal ideal m. For an A-module M, $\mu^i(m, M)$ is called the i-th *Bass number* of M and denoted by $\mu^i(M)$. Let M be a finitely generated A-module of dimension s. The *type* of M, denoted by $r(M)$, is defined to be $\mu^s(M)$. Let p be in $\mathrm{Supp}(M)$. If $\dim M_p + \dim A/p = \dim M$, then $r(M_p) \leq r(M)$ by [4, Theorem (5.1)] or [8, Proposition II. 4.1]. We here note that if A is (S_2) and

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