18. Equidimensional Toric Extensions of Symplectic Groups

By Haruhisa NAKAJIMA

Deparment of Mathematics, Keio University

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§ 0. G (resp. T) will always stand for a connected reductive complex algebraic group (resp. connected complex algebraic torus). We will use any of the notations ρ, (ρ, G) or (V, G) to denote a finite dimensional representation $\rho : G \to GL(V)$ over the complex number field $\boldsymbol{C}$ and often confuse ρ with the affine space V. An algebraic action of G on an affine variety X (abbr. (X, G)) is said to be *cofree* (resp. *equidimensional*), if $\boldsymbol{C}[X]$ is $\boldsymbol{C}[X]^G$-free (resp. if $X \to X/G$ is equidimensional), where $\boldsymbol{C}[X]$ denotes the affine coordinate ring of X and X/G denotes the algebraic quotient of X. On the other hand, (X, G) is said to be *stable*, if X contains a non-empty open subset consisting of closed G-orbits. For toric actions, we have proved in [5] the following result, which is fundamental in this paper:

Theorem 0.1 ([5]). *Let X be an affine conical factorial variety with an algebraic action of T compatible with the conical structure of X. Let W be a dual of a homogeneous T-submodule of $\boldsymbol{C}[X]$ which minimally generates $\boldsymbol{C}[X]$ as a $\boldsymbol{C}$-algebra. Then (X, T) is stable and equidimensional if and only if so is (W, T). If these conditions are satisfied, then both actions (X, T) and (W, T) are cofree.*

V. L. Popov and V. G. Kac conjectured that equidimensional representations are cofree. Concerning their conjecture, we will obtain

Theorem 0.2. *Suppose that the commutator subgroup of G is symplectic and of rank ≥ 3. Then finite dimensional equidimensional stable representations of G are cofree.*

We denote by G' the commutator subgroup of G and say that (V, G) is *relatively equidimensional* (resp. *relatively stable*), if $(V/G', G/G')$ is equidimensional (resp. stable). The purpose of this paper is to show

Theorem 0.3. *Under the same circumstances as in* (0.2), *suppose that the natural action of $Z(G)^0$ on $V/V^{G'}$ is nontrivial. If (V, G) is relatively stable and relatively equidimensional, then the restriction of (V, G) to G' (i.e., $((V, G), G')$) is cofree.*

This assertion does not hold, in the case where the semisimple rank of G is ≤ 2 (cf. [4]). Since equidimensional (resp. stable) representations are relatively equidimensional (resp. relatively stable), (0.2) follows from this and the classification [1] obtained by O. M. Adamovich and G. W. Schwarz. Some (calculative) parts of our proofs are left to the readers. The related study on

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