# 75. *Hilbert Spaces of Analytic Functions Associated with Generating Functions of Spherical Functions on U(n)/U(n−1)*

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**1. Introduction.** Let $\boldsymbol{R}$ or $\boldsymbol{C}$ be the field of real or complex numbers, $S(\boldsymbol{R}^n)$ or $S(\boldsymbol{C}^n)$ the unit sphere in $\boldsymbol{R}^n$ or $\boldsymbol{C}^n$ and $x \mapsto \bar{x}$ the usual conjugation in $\boldsymbol{C}$.

We denote by $F$ the Hilbert space of analytic functions $f(w)$ of $n$ complex variables $w = {}^t(w_1, w_2, \ldots, w_n) \in \boldsymbol{C}^n$, with the inner product defined by

$$(f, g) = \pi^{-n} \int_{C^n} \overline{f(w)} g(w) \exp(-|w_1|^2 - \cdots - |w_n|^2) dw_1 \cdots dw_n,$$

where

$$dw_1 \cdots dw_n = du_1 \cdots du_n dv_1 \cdots dv_n,\ w_j = u_j + iv_j (u_j, v_j \in \boldsymbol{R}),$$

and by $H$ the usual Hilbert space $L^2(\boldsymbol{R}^n)$.

V. Bargmann constructed in [1] a unitary mapping $A$ from $H$ onto $F$ given by an integral operator whose kernel is considered as a generating function of the Hermite polynomials. More precisely, $f = A\phi$ for $\phi \in H$ is defined by

$$f(w) = \int_{R^n} A(w, t) \phi(t) d^n t,$$

where

$$A(w, t) = \pi^{-n/4} \prod_{j=1}^{n} \exp\left\{-\frac{1}{2}(w_j^2 + t_j^2) + 2^{1/2} w_j t_j\right\}.$$

On the other hand, in [5] we showed that similar constructions are possible for the Gegenbauer polynomials $C_m^\lambda$, $m = 0,1,2,\ldots$, which essentially give the zonal spherical functions on the homogeneous space $SO(n)/SO(n-1) \cong S(\boldsymbol{R}^n)$. That is to say, let $F_\lambda$ be the Hilbert space of analytic functions $f$ of one complex variable on the unit disk $B$ in $\boldsymbol{C}$, with the inner product given by

$$\langle f, g \rangle_\lambda = \int_B \overline{f(w)} g(w) \rho_\lambda(|w|^2) dudv \quad (w = u + iv,\ u, v \in \boldsymbol{R}),$$

where

$$\rho_\lambda(t) = \begin{cases} \dfrac{1}{\Gamma(2\lambda - 1)} t^{\lambda-1} \displaystyle\int_t^1 s^{-\lambda}(1-s)^{2\lambda-2} ds & (\lambda > 1/2) \\ t^{\lambda-1}\left\{\dfrac{\Gamma(1-\lambda)}{\Gamma(\lambda)} - \dfrac{1}{\Gamma(2\lambda-1)} \displaystyle\int_0^t s^{-\lambda}(1-s)^{2\lambda-2} ds\right\} & (0 < \lambda \leq 1/2), \end{cases}$$

and let $K_\lambda$ be the usual $L^2$ space on the open interval $(-1,1)$ with respect to the measure $(1 - x^2)^{\lambda - 1/2} dx$. Then we have the following proposition (cf.