# 2. Galois Subfields of Abelian Function Field of Two Variables

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(Communicated by Heisuke HIRONAKA, M. J. A., Jan 12, 1994)

Let $L$ be an abelian function field of two variables over $\boldsymbol{C}$, and $K$ be a Galois subfield of $L$, i.e., $L$ is a finite algebraic Galois extension of $K$. We classify such $K$ by a suitable complex representation of the Galois group $G = Gal(L/K)$.

Let $A$ be the abelian surface with the function field $L$. Since $g \in G$ induces an automorphism of $A$, we have a complex representation $gz = M(g)z + t(g)$, where $M(g) \in GL_2(\boldsymbol{C})$, $z \in \boldsymbol{C}^2$, and $t(g) \in \boldsymbol{C}^2$. Fixing the representation, we put $G_0 = \{g \in G \mid M(g) = 1_2\}$, $H = \{M(g) \mid g \in G\}$ and $H_1 = \{M(g) \in H \mid det\, M(g) = 1\}$. Then we have the following exact sequences of groups:

$$1 \to G_0 \to G \to H \to 1,$$
$$1 \to H_1 \to H \xrightarrow{d} C_n \to 1,$$

where $d(M(g)) = detM(g)$, and $H/H_1$ is a cyclic group $C_n$ of order $n \leq 12$. The quotient surface $A/G_0$ is also an abelian surface. Note that the function field of the surface $A/G$ is isomorphic to $K$.

**Definition.** We call $H$ a holonomy part of the complex representation of $G$.

The holonomy part is completely determined by Fujiki [1], in which he studies automorphisms fixing the origin. By a slightly different method from his, i.e., by considering Sylow groups of $H$, we can show the following readily.

**Proposition 1.** *The order of $H$ is 5, 10 or $2^a \cdot 3^b$, where $a \leq 5$ and $b \leq 2$.*

Since the commutative group $G_0$ is a normal subgroup of $G$, we have

**Corollary 2.** *The Galois group G is solvable.*

The main purpose of this note is to classify $K$ by using the holonomy part. But we have no suitable language in the category of fields, so we do the classification in the equivalent category, i.e., using the language of the birational classification of algebraic surfaces. Note that in the case of elliptic curve $E$ the similar classification is simple, i.e., $E/G$ is rational if and only if $H$ is not trivial.

Let $[x, y]$ denote the diagonal matrix $\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$ and $e_n$ denote $exp(2\pi \sqrt{-1}/n)$. Then we have

**Lemma 3.** *If $H$ contains $[e_n, e_n]$, where $n = 3, 4, 6$, then $A$ is isomorphic to $E \times E$, where $E = C/(1, e_n)$.*

Since each $M \in H$ defines also an automorphism of $A$, the quotient