8. *On Rational Approximations to Linear Forms in Values of G-functions*

By Makoto NAGATA

Department of Mathematics, Tokyo Institute of Technology

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C. L. Siegel [8] defined E-functions and G-functions. They are solutions of linear differential equations which are expressed as Taylor series $\sum_{m=0}^{\infty} a_m x^m$ with coefficients in an algebraic number field.

E-functions satisfy

(1) for any $\varepsilon > 0$, the absolute values of $m!a_m$ and its conjugates do not exceed $Cm^{\varepsilon m}$,

(2) there is a sequence of common denominator d_m for $a_0, a_1, 2!a_2, \ldots, m!a_m$ which does not exceed $Cm^{\varepsilon m}$.

And G-functions satisfy

(3) the absolute values of a_m and its conjugates do not exceed C^m,

(4) there is a sequence of common denominator d_m for $a_0, a_1, a_2, \ldots, a_m$ which does not exceed C^m.

(C is a sufficiently large positive constant which is independent of m.)

Siegel utilized E-functions to obtain some results in the theory of transcendental numbers and suggested that G-functions will be also useful for similar purposes. The theory of these functions has been developed by different authors. In particular, Shidlovskii [7] proved the transcendency of the values in E-functions using the classical Padé approximations. The purpose of this paper is to show that we can use the method of [7] to G-functions in making more precise the definition of Padé approximations to obtain the best possible bounds of irrationality measure for linear independence in values of G-functions in some cases.

We assume $f_i^{(j)}(x) \in \mathbf{Q}[[x]]$ have non zero radii of convergence at $x = 0$.

We have the following.

Theorem A. *Let $f_i(x)$ $(i = 1, \ldots, n)$ be a non zero solution of a scalar linear differential equation of the order m_i over $\mathbf{Q}(x)$:*

$$\left(\frac{d}{dx}\right)^{m_i} f_i(x) + a_{m_i-1}^{(i)}(x)\left(\frac{d}{dx}\right)^{m_i-1} f_i(x) + \cdots + a_0^{(i)}(x) f_i(x) = 0,$$

where $a_j^{(i)}(x) \in \mathbf{Q}(x)$ $(i = 1, \ldots, n; j = 0, \ldots, m_i - 1)$. Put $f_i^{(j)}(x) := \left(\frac{d}{dx}\right)^j f_i(x)$ and $m := \sum_{i=1}^{n} m_i$. Suppose $f_i^{(j)}(x)$ $(i = 1, \ldots, n; j = 0, \ldots, m_i - 1)$ are linearly independent over $\mathbf{Q}(x)$. Let ε_0 be fixed in $\frac{1}{2} > \varepsilon_0 > 0$. Then there are effective constants C_1, depending only on $f_i^{(j)}(x)$ $(i = 1, \ldots, n; j = 0, \ldots, m_i - 1)$, ε_0, m and C_2, depending only on $f_i^{(j)}(x)$ $(i = 1, \ldots, n; j = 0, \ldots, m_i - 1)$, ε_0, m, r such that the following holds.

Let r be any rational number