37. An Extension of the Derivative of Meromorphic Functions to Holomorphic Curves

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1. Introduction. A holomorphic curve from $\boldsymbol{C}$ into $P^n(\boldsymbol{C})$ has no notion which plays exactly the same role as the derivative of meromorphic functions. Our purpose of this paper is then to introduce a sort of derivative to holomorphic curves which possesses similar properties to the derivative of meromorphic functions.

Let $f : \boldsymbol{C} \to P^n(\boldsymbol{C})$ be a holomorphic curve and let

$$(f_1, \dots, f_{n+1}) : \boldsymbol{C} \to \boldsymbol{C}^{n+1} - \{0\}$$

be a reduced representation of f, where n is a positive integer. Then, $f_1, \dots, f_{n+1}$ are entire functions without common zeros for all. The characteristic function $T(r, f)$ of f is defined as follows:

$$T(r, f) = \frac{1}{2\pi} \int_0^{2\pi} \log \| f(re^{i\theta}) \| \, d\theta - \log \| f(0) \|, \tag{1}$$

where $\| f(z) \| = (| f_1(z) |^2 + \cdots + | f_{n+1}(z) |^2)^{1/2}$. In addition, put

$$U(z) = \max_{1 \leq j \leq n+1} | f_j(z) |, \tag{2}$$

then we have the relation

$$T(r, f) = \frac{1}{2\pi} \int_0^{2\pi} \log U(re^{i\theta}) d\theta + O(1) \qquad ([1]). \tag{3}$$

It is said that f is transcendental if $\lim_{r\to\infty} T(r, f) / \log r = \infty$. We denote by $\rho(f)$ the order of f:

$$\rho(f) = \limsup_{r\to\infty} \frac{\log T(r, f)}{\log r}$$

and by $S(r, f)$ any quantity satisfying

$$S(r, f) = \begin{cases} O(\log r) \ \ (r \to \infty) & \text{if } \rho(f) < \infty, \\ O(\log rT(r, f)) \ (r \to \infty, \ r \notin E) & \text{if } \rho(f) = \infty, \end{cases}$$

where E is a subset of $[0, \infty)$ for which $m(E) < \infty$.

From now on throughout the paper we suppose that f is non-degenerate; that is to say, the functions $f_1, \dots, f_{n+1}$ are linearly independent over $\boldsymbol{C}$.

Let $W(f_1, \dots, f_{n+1})$ be the Wronskian of $f_1, \dots, f_{n+1}$. Then, it is well-known that $f_1, \dots, f_{n+1}$ are linearly independent over $\boldsymbol{C}$ if and only if $W(f_1, \dots, f_{n+1})$ is not identically equal to zero.

Our definition of an extension of the derivative of meromorphic functions to non-degenerate holomorphic curves is as follows.

Definition (extension of the derivative). we call the holomorphic curve induced by the mapping