31. *Crepant Resolution of Trihedral Singularities*

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§1. Introduction. The purpose of this paper is to construct a crepant resolution of quotient singularities by finite subgroups of $SL(3, \mathbf{C})$ of certain type, and prove that each Euler number of the minimal model is equal to the number of conjugacy classes.

The problem of finding a nice resolution of quotient singularities by finite subgroups of $SL(3, \mathbf{C})$ arose from mathematical physics. In the superstring theory, the dimension of the space-time is 10, four of them are usual space and time dimensions, and other six are compactified on a compact Calabi-Yau space M. From a point of view of algebraic geometry, the Calabi-Yau space is a smooth three-dimensional complex projective variety whose canonical bundle is trivial and fundamental group is finite.

In the physics of superstring theory, one considers the string propagation on a manifold M which is a quotient by a finite subgroup of symmetries G. By a physical argument of string vacua of M/G, one concludes that the correct Euler number for the theory should be the "orbifold Euler characteristic" [3], defined by

$$\chi(M, G) = \frac{1}{|G|} \sum_{gh=hg} \chi(M^{\langle g,h \rangle}),$$

where the summation runs over all pairs of commuting elements of G, and $M^{\langle g,h \rangle}$ denotes the common fixed set of g and h. For the physicist's interest, we only consider M whose quotient space M/G has trivial canonical bundle.

Conjecture I ([3]). *There exists a resolution of singularities* $\widetilde{M/G}$ *s.t.* $\omega_{\widetilde{M/G}} \simeq \mathcal{O}_{\widetilde{M/G}}$, *and*

$$\chi(\widetilde{M/G}) = \chi(M, G).$$

This conjecture follows from its local form [6]:

Conjecture II (local form). *Let* $G \subset SL(3, \mathbf{C})$ *be a finite group. Then there exists a resolution of singularities* $\sigma : \tilde{X} \to \mathbf{C}^3/G$ *with* $\omega_{\tilde{X}} \simeq \mathcal{O}_{\tilde{X}}$ *and*

$$\chi(\tilde{X}) = \#\{\text{conjugacy class of } G\}.$$

In algebraic geometry, the conjecture says that a minimal model of the quotient space by a finite subgroup of $SL(3, \mathbf{C})$ is non-singular.

Conjecture II was proved for abelian groups by Roan ([18]), and independently by Markushevich, Olshanetsky and Perelomov ([11]) by using toric method. It was also proved for 5 other groups, for which X are hypersurfaces: (i) WA_3^+, WB_3^+, WC_3 where WX^+ denotes the positive determinant part of the Weyl group WX of a root system X by Bertin and Markushevich ([1]), (ii) H_{168} by Markushevich ([10]), and (iii) I_{60} by Roan ([19]).