4. *Kähler Magnetic Fields on a Complex Projective Space*

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In this note we study trajectories of charged particles under the action of a Kähler magnetic field, a magnetic field corresponding to the Kähler form, on a complex projective space. We show that they are small circles on a totally geodesic embedded 2-dimensional sphere.

A magnetic field on a complete Riemannian manifold M is a closed 2-form B. Let $\Omega = \Omega_{\boldsymbol{B}} : TM \to TM$ denote the skew symmetric operator on the tangent bundle satisfying $\boldsymbol{B}(X, Y) = \langle X, \Omega(Y)\rangle$. We call a curve γ on M a trajectory for this magnetic field if it is a solution of the equation $\nabla_{\dot\gamma}\dot\gamma = \Omega(\dot\gamma)$. Every trajectory γ has constant speed because $\frac{d}{dt}\|\dot\gamma(t)\|^2 = 2\langle\Omega(\dot\gamma(t)), \dot\gamma(t)\rangle = 0$. If γ is a trajectory of constant speed c for a magnetic field $\boldsymbol{B}$, the curve $\sigma(t) = \gamma(t/c)$ is a trajectory of unit speed for the magnetic field $c^{-1}\boldsymbol{B}$. We may therefore suppose trajectories are parametrized by their arc-length.

A magnenic field is called *uniform* if the associated skew symmetric operator is parallel $\nabla\Omega = 0$. Typical examples of uniform magnetic fields are scalar multiples of the volume form k-dvol on Riemann surfaces. On surfaces of constant curvature trajectories of such magnetic fields are well-known. On a sphere trajectories are small circles, on a Euclidean plane they are circles (in usual sense), and they are all closed. On a hyperbolic plane the feature is quite different. When the strength $|k|$ is greater than 1, trajectories are closed. But when it is not greater than 1 they are open (see [2] and also [5]).

We here give another example of uniform magnetic fields. Let (M, J) be a Kähler manifold and $\boldsymbol{B}_J$ denote the Kähler form; $\boldsymbol{B}_J(X, Y) = \langle X, JY\rangle$. Then the closed 2-form $\boldsymbol{B} = k\boldsymbol{B}_J$ with constant k is a uniform magnetic field. We shall call such field a *Kähler magnetic field.* It is quite natural to study trajectories for Kähler magnetic fields on manifolds of constant holomorphic sectional curvature. Trivially we can conclude that trajectories for a Kähler magnetic field are congruent on a manifold of constant holomorphic sectional curvature. That is, for given two trajectories γ and σ (of unit speed) for a Kähler magnetic field, we have a holomorphic isometry φ with $\sigma = \varphi\circ\gamma$.

In this note we show an explicit expression of trajectories for Kähler magnetic fields on a complex projective space. Let $\pi : S^{2n+1} \to \boldsymbol{CP}^n$ denote the Hopf fibration of a standard sphere onto a complex projective space. The tangent space of $\boldsymbol{CP}^n$ at $\pi(x)$ can be identified with the horizontal subspace