27. *On Hermitian Eisenstein Series*

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In [8], Shimura studied the analytic nature of the Eisenstein series for the Hermitian modular group. The main purpose of this note is to add some results in the theory of Hermitian Eisenstein series of low weights. The detailed proof will be given elsewhere.

1. Hermitian Eisenstein series. Let G_n be the group defined by

$$G_n = \{M \in SL_{2n}(\boldsymbol{C}) \mid {}^t\bar{M}J_nM = J_n\},\ J_n = \begin{pmatrix} 0 & E_n \\ -E_n & 0 \end{pmatrix}.$$

We write a typical element M of G_n in the form $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ with matrices A, B, C, D of size n. Denote by P_n the subgroup of G_n consisting of the element M for which $C = 0$. Let $\boldsymbol{H}_n$ be the domain defined by

$$\boldsymbol{H}_n = \{Z \in M_n(\boldsymbol{C}) \mid I(Z) := (2i)^{-1}(Z - {}^t\bar{Z}) > 0\}.$$

This is called the *Hermitian upper half space of degree n*. The group G_n acts on $\boldsymbol{H}_n$ as

$$Z \mapsto M\langle Z\rangle = (AZ+B)(CZ+D)^{-1},\ M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in G_n.$$

Let $\boldsymbol{K}$ be an imaginary quadratic number field of discriminant $d_{\boldsymbol{K}}$. We denote by $\mathcal{O}_{\boldsymbol{K}}$ the ring of integers in $\boldsymbol{K}$. The *Hermitian modular group of degree n over* $\boldsymbol{K}$ is defined by $\Gamma_n(\boldsymbol{K}) := G_n \cap M_{2n}(\mathcal{O}_{\boldsymbol{K}})$. We consider the following Eisenstein series:

$$E_k^{(n)}(Z, s) = \sum_{\begin{pmatrix} A & B \\ C & D \end{pmatrix} : \Gamma_n(\boldsymbol{K})_0 \backslash \Gamma_n(\boldsymbol{K})} \det(CZ+D)^{-k} \,|\det(CZ+D)|^{-s},$$

where $(Z, s) \in \boldsymbol{H}_n \times \boldsymbol{C}$, $k \in 2\boldsymbol{Z}$ and $\Gamma_n(\boldsymbol{K})_0 = P_n \cap \Gamma_n(\boldsymbol{K})$. It is known that this series is absolutely convergent for $\mathrm{Re}(s) + k > 2n$ (cf. Braun [2]). Moreover, it can be continued as a meromorphic function in s to the whole complex plane (e.g. cf. Shimura [8]). We shall call this the *Hermitian Eisenstein series for* $\Gamma_n(\boldsymbol{K})$.

2. Functional equation. We introduce a functional equation of $E_k^{(n)}(Z, s)$, which plays an important role in the proof of our main result (Theorem 3). We denote by $\chi_{\boldsymbol{K}}$ the Kronecker symbol of $\boldsymbol{K}$. For $m \in \boldsymbol{Z}(m \geqq 0)$, we put

$$\rho(s\,;\chi_K^m) := \begin{cases} \pi^{-\frac{s}{2}}\Gamma(s/2)\zeta(s) & \text{if } m \text{ is even,} \\ |d_{\boldsymbol{K}}|^{\frac{s}{2}}\pi^{-\frac{s}{2}}\Gamma((s+1)/2)L(s\,;\chi_{\boldsymbol{K}}) & \text{if } m \text{ is odd,} \end{cases}$$

where $\Gamma(s)$ is the gamma function, $\zeta(s)$ is the Riemann zeta function, and $L(s\,;\chi_{\boldsymbol{K}})$ is the Dirichlet L-function with respect to $\chi_{\boldsymbol{K}}$. It is known that, for each m, the meromorphic function $\rho(s\,;\chi_{\boldsymbol{K}}^m)$ satisfies a functional equation $\rho(1-s\,;\chi_{\boldsymbol{K}}^m) = \rho(s\,;\chi_{\boldsymbol{K}}^m)$. Introduce a polynomial $\varepsilon_n(s) \in \boldsymbol{Z}[s]$ by $\varepsilon_n(s) = \Pi_{m=0}^{n-1}(s-m)$. Our first result is as follows: