24. On Confluences of the General Hypergeometric Systems

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Introduction. Let r and $n(>r)$ be positive integers and let $Z_{r,n}$ be the set of $r \times n$ complex matrices of maximal rank.

In the preceding paper [7], we introduced, for any given composition $\lambda = (\lambda_1, \dots, \lambda_l)$ of the integer n, the generalized confluent hypergeometric functions with variables $z = (z_{ip})_{0 \le i \le r-1, 0 \le p \le n-1} \in Z_{r,n}$. They are defined as solutions of the system of partial differential equations on $Z_{r,n}$ called the generalized confluent hypergeometric system (see Definition 1.1). In case where the composition of n is $\lambda = (1, \dots, 1)$, the generalized confluent hypergeometric functions coincide with the general hypergeometric functions due to K. Aomoto and I. M. Gelfand ([1], [2]).

One may ask why we have given the name "the generalized confluent" hypergeometric functions to the functions we introduced. The purpose of this paper is to justify our naming to these functions. In fact we show that the generalized confluent hypergeometric systems can be obtained from the Aomoto-Gelfand's system by a finite number of certain limit processes called the processes of confluence (see Theorem 2.5). It is to be noted that the processes of confluence for our systems are determined from the group theoretic point of view (Theorems 2.3, 2.4).

To explain our problem more concretely, we recall a classical example: a confluence of two singular points for the hypergeometric equation of Gauss. The Gauss hypergeometric equation is

(0.1) $x(1-x)u'' + \{\gamma - (\alpha+\beta+1)x\}u' - \alpha\beta u = 0, \quad ' = d/dx.$

For the equation (0.1), consider a change of variables and parameters

$$x = \varepsilon\xi, \quad \beta = 1/\varepsilon.$$

Then the equation for (ξ, u) is

$$\xi(1-\varepsilon\xi)\frac{d^2u}{d\xi^2} + (\gamma - \varepsilon(\alpha + \varepsilon^{-1} + 1)\xi)\frac{du}{d\xi} - \alpha u = 0 \tag{0.2}$$

and the coefficients of $d^2u/d\xi^2$, $du/d\xi$ and u depend holomorphically on ε at $\varepsilon = 0$. Putting $\varepsilon = 0$ in the equation (0.2) we obtain the Kummer's confluent hypergeometric equation

$$\xi\frac{d^2u}{d\xi^2} + (\gamma - \xi)\frac{du}{d\xi} - \alpha u = 0. \tag{0.3}$$

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