91. *On the Region Free from the Poles of the Resolvent for the Elastic Wave Equation with the Neumann Boundary Condition*

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1. Introduction. Let Ω be an exterior domain in $\boldsymbol{R}^n (n \geq 3)$ with smooth and compact boundary Γ. We consider the isotropic elastic wave equation with the Neumann boundary condition

$$\text{(N)} \qquad \begin{cases} (A(\partial_x) - \partial_t^2)u(t, x) = 0 & \text{in } \boldsymbol{R} \times \Omega, \\ N(\partial_x)u(t, x) = 0 & \text{on } \boldsymbol{R} \times \Gamma, \\ u(0, x) = f_0(x),\ \partial_t u(0, x) = f_1(x) & \text{on } \Omega, \end{cases}$$

where $u(t, x) = {}^t(u_1(t, x), \cdots, u_n(t, x))$ is the displacement vector. Using the stress tensor $\sigma_{ij}(u) = \lambda(\operatorname{div} u)\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ and the unit outer normal vector $\nu(x) = {}^t(\nu_1(x), \nu_2(x), \cdots, \nu_n(x))$ to Ω at $x \in \Gamma$, we can give the operator $A(\partial_x)$ and the boundary operator $N(\partial_x)$ by $(A(\partial_x)u)_i = \sum_{j=1}^n \partial_{x_j}(\sigma_{ij}(u))$, $(N(\partial_x)u)_i = \sum_{j=1}^n \nu_j(x)\sigma_{ij}(u)\,|_\Gamma (i = 1, 2, \cdots, n)$. Note that $A(\partial_x)$ can also be written as $A(\partial_x)u = \mu\Delta u + (\lambda + \mu)$ grad $(\operatorname{div} u)$.

We assume that the Lamé constants λ and μ are independent of the variables t and x and satisfy

$$\lambda + \frac{2}{n}\mu > 0 \text{ and } \mu > 0.$$

We define the outgoing resolvent $R(z)$ of the problem (N) as the solution operator of the reduced elastic wave equation

$$\begin{cases} (A(\partial_x) + z^2)v(x\,; z) = f(x) & \text{in } \Omega, \\ N(\partial_x)v(x\,; z) = 0 & \text{on } \Gamma, \\ v(x\,; z) \text{ is outgoing,} \end{cases}$$

where the word "outgoing" means that $v(x\,; z)$ is the $L^2(\Omega)$-solution if $\operatorname{Im} z < 0$ and the analytic continuation of the $L^2(\Omega)$-solution in the region $\operatorname{Im} z < 0$ if $\operatorname{Im} z \geq 0$. Note that for any $a > 0$ with $\Gamma \subset B_a = \{x \in \boldsymbol{R}^n \,||\, x| < a\}$, $R(z)$ is a $B(L_a^2(\Omega), H^2(\Omega \cap B_a))$-valued meromorphic function in $\tilde{C}$, and a $B(L_a^2(\Omega), H^2(\Omega \cap B_a))$-valued holomorphic function in $\operatorname{Im} z \leq 0$, $z \neq 0$, where $L_a^2(\Omega) = \{f \in L^2(\Omega) \mid \operatorname{supp} f \subset \Omega \cap B_a\}$, and $\tilde{C} = C$ if n is odd, $\tilde{C} = \left\{z \in C \setminus \{0\} \mid -\frac{3}{2}\pi < \arg z < \frac{1}{2}\pi\right\}$ if n is even (cf. Iwashita and Shibata [3]).

The purpose of this note is to give some information about the location of the poles of the outgoing resolvent of the problem (N). For the problem (N), it is well known that there exists the Rayleigh surface wave propagat-