86. *Recurrence of a Diffusion Process in a Multidimensional Brownian Environment**)

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Introduction. Let $\boldsymbol{W}$ be the space of continuous functions on $\boldsymbol{R}^d$ vanishing at the origin. In this paper an element of $\boldsymbol{W}$ is called an environment. Given an environment W, we consider a diffusion process $\boldsymbol{X}_W = \{X(t),\ t \geq 0,\ P_W^x,\ x \in \boldsymbol{R}^d\}$ with generator

$$\frac{1}{2}(\Delta - \nabla W \cdot \nabla) = \frac{1}{2} e^W \sum_{k=1}^{d} \frac{\partial}{\partial x_k}\left(e^{-W} \frac{\partial}{\partial x_k}\right).$$

When W is bounded, the result of Nash [8] for fundamental solutions of parabolic equations guarantees the existence of a diffusion process $\boldsymbol{X}_W^0$ with generator

$$\sum_{k=1}^{d} \frac{\partial}{\partial x_k}\left(e^{-W} \frac{\partial}{\partial x_k}\right).$$

For a general W we still have a nice diffusion process $\boldsymbol{X}_W^0$ (e.g. see [4]) and hence $\boldsymbol{X}_W$ can be constructed from $\boldsymbol{X}_W^0$ through a random time change. Without any assumption on the behavior of $W(x)$ for large $|x|$ the process $\boldsymbol{X}_W$ may explode within a finite time, but such a case is excluded automatically since we are interested in the recurrence of $\boldsymbol{X}_W$. We consider the probability measure $\boldsymbol{P}$ on $\boldsymbol{W}$ with respect to which $\{W(x),\ x \in \boldsymbol{R}^d,\ P\}$ is a Lévy's Brownian motion with a d-dimensional time. The collection of diffusion processes $\boldsymbol{X} = \{\boldsymbol{X}_W\}$ in which W is allowed to vary as a random element in $(\boldsymbol{W}, P)$ is called a diffusion in a d-dimensional Brownian environment. When $d = 1$ this was considered by Brox [1] and Schumacher [9] as a diffusion model exhibiting the same asymptotic behavior as Sinai's random walk in a random environment ([10]); see also [11] for some refined results. Recently Mathieu [7] obtained some very interesting results concerning a long time asymptotic problem for $\boldsymbol{X}$ in the case $d \geq 2$. Motivated by [7] the present paper was written.

In this paper we prove that $\boldsymbol{X}_W$ is recurrent for almost all Brownian environments W in any dimension d, namely, for any nonnegative Borel function f on $\boldsymbol{R}^d$ such that $f > 0$ on a set of positive Lebesgue measure the equality

$$P_W^x\left\{\int_0^\infty f(X(t))\,dt = \infty\right\} = 1,\ x \in \boldsymbol{R}^d,$$

holds for almost all W with respect to $\boldsymbol{P}$. In [3] Fukushima, Nakao and Takeda discussed the same problem but with the replacement of $W(x)$ by $\tilde{W}(|x|)$,

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