82. *Spined Products of Some Semigroups**)

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Spined products of semigroups were first defined and studied by N. Kimura, 1958, [7]. After that, spined products have been considered many a time, predominantly those of a band and a semilattice of semigroups with respect to their common semilattice homomorphic image. Spined and subdirect products of a band and a semilattice of groups are studied by M. Yamada [13], [14], J. M. Howie and G. Lallement [6] and by M. Petrich [10]; spined products of a band and some types of semilattices of monoids are studied by F. Pastijn [8], A. El-Qallali [3], [4], and by R. J. Warne [12]. For other considerations of these products, we refer to [4], [5], [7], [9], [15]. In the quoted papers, spined products are considered in connection with some types of bands of semigroups. In this paper, we give a general composition for bands of semigroups that are (punched) spined products of a band and a semilattice of semigroups. This composition, in some sense, is a generalization of a well-known semilattice composition (see Theorem III 7.2. [9]).

Let B be a band. By $\leq_1$ and $\leq_2$ we denote quasi-orders on B defined by $i \leq_1 j \Leftrightarrow ij = j$, $i \leq_2 j \Leftrightarrow ji = j$, and by $\leq$ we denote the *natural order* on B defined by "$i \leq j$ means that $i \leq_1 j$ and $i \leq_2 j$". For $i \in B$, we will denote by $[i]$ the class of an element i in the greatest semilattice decomposition of a band B (so $[i]$ is an element of the greatest semilattice homomorphic image of B). If S is a band B of semigroups S_i, $i \in B$, then for $k \in B$, F_k will denote the semigroup $F_k = \cup \{S_i \mid i \in B,\ [i] \geq [k]\}$. If θ is a homomorphism of a semigroup S into a semigroup S', and if T is a common subsemigroup of S and S', then θ is a *T-homomorphism* if $a\theta = a$, for all $a \in T$. A subsemigroup T of a semigroup S is a *retract* of S if there exists a homomorphism θ of S onto T such that $a\theta = a$, for all $a \in T$. We call such a homomorphism a *retraction*. If T is a subsemigroup of a semigroup S, then we say that S is an *oversemigroup* of T. If ρ is a congruence on a semigroup S, then we denote by $\rho^\natural$ the natural homomorphism of S onto S/ρ. If P and Q are two semigroups having a common homomorphic image Y, then the *spined product of P and Q with respect to Y* is $S = \{(a, b) \in P \times Q \mid a\varphi = b\psi\}$, where $\varphi : P \to Y$ and $\psi : Q \to Y$ are homomorphisms onto Y. If Y is a semilattice and P and Q are a semilattice Y of semigroups P_α, $\alpha \in Y$, and Q_α, $\alpha \in Y$, respectively, then the spined product of P and Q with respect to Y is $S = \cup_{\alpha \in Y} P_\alpha \times Q_\alpha$. A subsemigroup S of a spined product of semigroups P and Q with respect to Y, that is also a subdirect product of P and Q, is a *punched spined product of P and Q with respect to Y*.

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