

81. A Table of the Dimensions of the Hilbert Modular Type Cusp Forms for the Hurwitz-Maass Extensions^{*)}

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1. Introduction and the table. For a square-free positive number D , let k be a real quadratic number field $\mathbf{Q}(\sqrt{D})$, and \mathfrak{o} the ring of integers in k . There is a unique maximal discrete extension of the Hilbert modular group $G = SL_2(\mathfrak{o})$ in $PL_2^+(\mathbf{R})^2$, which is called the *Hurwitz-Maass extension* G_m of G . G_m acts properly discontinuously on H^2 (H being a complex upper half plane). We consider the spaces $S(D)_m$ of the cusp forms of weight two with respect to G_m in H^2 .

For the ordinary Hilbert modular group G and the extended Hilbert modular group \hat{G} , we have already given dimension tables in [5], [6]. This note continues the work of [6]: we tabulate the dimensions of $S(D)_m$ for a square-free D , $1 < D < 1000$, with a computer assistance. In the following table, the number D is given by

$$(1) \quad D = i + j \quad (i = \text{row number}, j = \text{column number}).$$

When the mark ‘-’ appears after a figure, $\mathbf{Q}(\sqrt{D})$ has a unit of negative norm. The mark ‘* *’ means that D is not square-free.

2. The method of the computation. For a square-free divisor w of the discriminant d_k of k , we denote by G_w the subgroup of G_m arising from the set of matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $a, b, c, d \in (w)^{1/2}$, $ad - bc = w$, where $(w)^{1/2}$ is an ideal whose square equals (w) . Then we have

$$(2) \quad G_m = \bigcup_{w|d_k} G_w.$$

Let t be a number of distinct primes dividing d_k . When $t = 1$, G_m coincides with G ; when $t = 2$ and k has no units of negative norm, G_m coincides with \hat{G} . G_m is also the maximal discrete subgroup of $PL_2^+(\mathbf{R})^2$ containing \hat{G} ([7]). The elliptic fixed points of G_m was investigated by Hausmann [1]. Note that $[G_m : G] = 2^{t-1}$ and G_m has $h^+ / 2^{t-1}$ inequivalent cusps where h^+ is the narrow class number of k .

By virtue of [1], [3], we get

Theorem. *The dimension of $S(D)_m$ is given by*

$$(3) \quad \dim S(D)_m = t_0 + t_1 + t_2 - 1.$$

Each term can be written as follows.

$$(4) \quad t_0 = (1/2^t) \zeta_k(-1)$$

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