80. The Schur Indices of the Irreducible Characters of $G_2(2^n)$

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Introduction. Let $G_2(q)$ be the finite Chevalley group of type (G_2) over a finite field $\boldsymbol{F}_q$ with q elements. It was shown in [3] that the following theorem holds for odd q:

Theorem. *The Schur index* $m_{\boldsymbol{Q}}(\chi)$ *of any complex irreducible character* χ *of* $G_2(q)$ *with respect to* $\boldsymbol{Q}$ *is equal to* 1.

In this paper, we shall prove that the theorem holds also for $q = 2^n$, as was announced in [3]. The complex irreducible characters of $G_2(2^n)$ have been calculated by the first named author and H. Yamada in [2]. In the following, $G_2(2^n)$ will be denoted simply by G.

Proof of the theorem for $q = 2^n$. For the notation of the conjugacy classes of $G = G_2(2^n)$, the characters of G, or of subgroups of G, etc., we follow those in [2].

Let B be the Borel subgroup of G and U its unipotent part. We first describe the character-values of the Gelfand-Graev character Γ_G of G and the induced character $1_U^G = \mathrm{Ind}_U^G(1_U)$; Γ_G is the character of G induced by the linear character of U given by $x_a(t_1)x_b(t_2)x_{a+b}(t_3) \cdots x_{3a+2b}(t_6) \to \phi(t_1)\phi(t_2)$, where ϕ is a previously fixed non-trivial additive character of $\boldsymbol{F}_{2^n}$. There are eight unipotent classes in $G : A_0, A_1, A_2, A_{31}, A_{32}, A_4, A_{51}$ and A_{52}; representatives of these classes are respectively: $h(1, 1, 1) = e$, $x_{3a+2b}(1)$, $x_{2a+b}(1)$, $x_{a+b}(1)x_{2a+b}(1)$, $x_{a+b}(1)x_{2a+b}(1)x_{3a+b}(\xi)$, $x_b(1)x_{2a+b}(1)x_{3a+b}(\eta)$, $x_a(1)x_b(1)$ and $x_a(1)x_b(1)x_{2a+b}(\xi)$. Then we have the following table:

	Γ_G	1_U^G
A_0	$(q^2-1)(q^6-1)$	$(q^2-1)(q^6-1)$
A_1	$-q^2+1$	$(q^2-1)(q^3-1)$
A_2	$-q^2+1$	$(q^2-1)(q-1)(2q+1)$
A_{31}	$-q^2+1$	$(q-1)^2(4q+1)$
A_{32}	$-q^2+1$	$(q-1)^2(2q+1)$
A_4	$-q^2+1$	$(q^2-1)(q-1)$
A_{51}	1	$(q-1)^2$
A_{52}	1	$(q-1)^2$

Since ϕ takes values in $\boldsymbol{Q}$, Γ_G is realizable in $\boldsymbol{Q}$. Also 1_U^G is clearly realizable in $\boldsymbol{Q}$.

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