10. *Prime Ideals in Noncommutative Valuation Rings in Finite Dimensional Central Simple Algebras*

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1. Introduction. In [2], Dubrovin introduced a notion of non-commutative valuation rings in simple Artinian rings, and proved some elementary properties of them. He obtained in [3] more detailed results concerning valuation rings in finite dimensional central simple algebras over fields.

In this paper, we investigate prime ideals in non-commutative valuation rings in the case of algebras. The key result is Proposition 9 which states that, for any ideal A of a valuation ring R, $\cap A^n$ is a prime ideal of R. Using this result, we characterize branched and unbranched prime ideals.

2. Throughout this paper, let V be a valuation domain with the quotient field K, and let R be a valuation ring in the sense of [2] in a finite dimensional central simple K-algebra Σ with its center V and $KR = \Sigma$.

First, we shall list the elementary properties of a non-commutative valuation ring R which are used frequently.

(A) R-ideals are linearly ordered by inclusion and the Jacobson radical $J(R)$ is the unique maximal ideal of R (§2 Theorem 4 (1) and §1 Theorem 4 of [2]).

(B) Each overring S of R is also a valuation ring, and $J(S)$ is a prime ideal of R (Theorem 4 (2) of [2, §2]).

(C) For any R-ideal A, $O_r(A) = O_l(A)$, where $O_r(A) = \{q \in \Sigma \mid Aq \subseteq A\}$, the *right order* of A, and $O_l(A) = \{q \in \Sigma \mid qA \subseteq A\}$, the *left order* of A (Corollary to Proposition 4 of [3, §2]).

(D) For any non-zero element $x \in R$, there is some regular $z \in R$ such that $RxR = zT = Tz$, where $T = O_r(RxR) = O_l(RxR)$ (Proposition 3 of [3, §2]).

(E) For any prime ideal P of R, $C(P) = \{c \in R \mid [c + P]$ is regular mod $P\}$ is a regular Ore set of R and so there exists the localization of R with respect to $C(P)$. We denote this by R_P. Let $p = P \cap V$. Then we have $R_P = R_p$, where R_p denotes the localization of R with respect to $V - p$ (Theorem 1 of [3, §2]).

(F) The mapping $P \to R_P$ is an inclusion reversing bijection between the set of prime ideals of R and the set of overrings of R. The inverse mapping is $S \to J(S)$. (Corollary to Theorem 4 of [2, §2] and Theorem 1 (3) of [3, §2].)

Now we shall investigate prime ideals of R. For any ideal A of R, we

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