78. *Gauss Decomposition of Connection Matrices and Application to Yang-Baxter Equation. II*

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(Communicated by Heisuke HIRONAKA, M. J. A., Oct. 12, 1993)

We follow the same terminologies as in [1].

1. Gauss decomposition of G. Case where $m = 2$. The matrix $G = G(x \mid \alpha_1)$ depends on x_2/x_1 and of size $n+1$. We denote by $g_{n-i,n-j} = g_{n-i,n-j}(x_2/x_1)$ its entries as

(1.1) $$g_{n-i,n-j} = (Y^+_{i,n-i} : \text{reg } Y^-_{j,n-j})_{\Phi^{(a)}_{n,2}}$$

where the corresponding summits $\xi = v^+_{i,n-i}$ and $\eta = v^-_{j,n-j}$ are given by $\xi_k = x_1 q^{1+(k-1)\gamma}$ $(1 \le k \le i)$, $x_2 q^{1+(k-i-1)\gamma}$ $(1+i \le k \le n)$ and $\eta_k = x_1 q^{-\beta-(k-1)\gamma}$ $(1 \le k \le j)$, $x_2 q^{-\beta-(k-i-1)\gamma}$ $(1+j \le k \le n)$ respectively.

First we present a few basic properties of the principal connection matrix G.

Lemma 1.

(1.2) $$\tau_1 G(x \mid \alpha_1) = {}^t G(x \mid \alpha_1) = S'^{-1}_{\tau_1} \cdot G(x \mid \alpha_1) \cdot S'_{\tau_1}$$

where ${}^t G(x \mid \alpha_1)$ denotes the transposed matrix and S'_{τ_1} denotes the matrix with only non-zero $(i, n-i)$th components $a_{i,n-i}\left(\frac{x_2}{x_1}\right)$,

$$a_{i,n-i}(u) = u^{-2\gamma i(n-i)} q^{\gamma^2 i(n-i)(-n+2i)+i(n-i)} \frac{\theta(q^{-i\gamma}u)_{\hat{i}}\,\theta(q^{(1-i)\gamma}u)_{\hat{i}}}{\theta(q^{1-(n-i)\gamma}u^{-1})_{\hat{i}}\,\theta(q^{1-(n-i-1)\gamma}u^{-1})_{\hat{i}}}$$

for $\hat{i} = \min(i, n-i)$. In particular $a_{0,n}(u) = a_{n,0}(u) = 1$.

(1.3) $$S'_{\tau_1} = \Lambda^{-1} S_{\tau_1} \tau_1 \Lambda, \text{ for } \Lambda = \mathrm{Diag}[\lambda_0, \dots, \lambda_n]$$

where $\lambda_i = \lambda_i(x_2/x_1) = \theta(q^{1-i\gamma}x_2/x_1)_{\hat{i}}\,\theta(q^{1-(i-1)\gamma}x_2/x_1)_{\hat{i}} \left(\frac{x_2}{x_1}\right)^{\hat{i}-\gamma i(n-i)} q^{i^2(n-i)\gamma^2 - i\hat{i}\gamma}$

and S_{τ_1} denotes the matrix with only non zero $(i, n-i)$th components 1 *so that* $S^2_{\tau_1} = 1$.

(1.4) $$G(x \mid \alpha_1)^{-1} = (q^{2n(\beta^2+\beta)}/(1-q)^{2n})\, M \cdot G(x^{-1} \mid -\alpha_1 - 2\beta + 2(n-1)(\gamma-1)) \cdot M'$$

where M and M' denote the diagonal matrices $M = \mathrm{Diag}[\mu_0, \dots, \mu_n]$, $M' = \mathrm{Diag}[\mu'_0, \dots, \mu'_n]$ *such that* $\mu_{n-i} = \left(\frac{x_2}{x_1}\right)^{2i\beta} a_i\left(\frac{x_2}{x_1}\right) a_{n-i}\left(\frac{x_1}{x_2}\right) a_{n-i,i}\left(\frac{x_2}{x_1}\right)$, $\mu'_{n-i} = \left(\frac{x_2}{x_1}\right)^{-2i\beta} a_i\left(\frac{x_1}{x_2}\right) a_{n-i}\left(\frac{x_2}{x_1}\right) a_{n-i,i}\left(\frac{x_1}{x_2}\right)$. *Here $a_i(u)$ denotes*

(1.5) $$a_i(u) = q^{i(i-1)\beta\gamma + \gamma^2(i-1)i(i+1)/3 + \gamma(i-1)i/2} \cdot \frac{\theta(q^{1+\gamma})_i\,\theta(q^{1+\beta})_i\,\theta(q^{1+\beta}u)_i}{\theta'(1)^i\,\theta(q^{1+\gamma})^i\,\theta(qu)_i},$$

where $\theta(u)_i$ denotes the product $\theta(u)\theta(uq^\gamma)\cdots\theta(uq^{(i-1)\gamma})$ and $\theta'(1) =$

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