77. 3-Sasakian Manifolds*)

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Introduction. In 1960 Sasaki [17] introduced a geometric structure related to an almost contact structure on a smooth manifold. This structure, which became known as a Sasakian structure, was studied extensively in the 1960's by an entire school of Japanese geometers (See [24] and references therein). In 1970 Kuo [14] refined this notion and introduced manifolds with Sasakian 3-structures. The same year Kuo and Tachibana, Tachibana and Yu, and Tanno [15, 22, 21] published foundational papers discussing Sasakian 3-structures and these structues were then vigorously studied by many Japanese mathematicians from 1970-1975. This intense analysis culminated with an important paper of Konishi [13] which shows the existence of a Sasakian 3-structure on a certain principal $SO(3)$ bundle over any quaternionic Kähler manifold of positive scalar curvature.

Earlier on, in 1973, Ishihara [10] had shown that if the distribution formed by the three Killing vector fields which define the Sasakian 3-structure is regular then the space of leaves is a quaternionic Kähler manifold. This fact led Ishihara to his foundational work on quaternionic Kähler manifolds [9]. Ishihara's and Konishi's observation that quaternionic Kähler and 3-Sasakian geometries are related is fundamental.

It is notable that in this early period the only examples of 3-Sasakian manifolds appearing in the literature were those of constant curvature, namely the spheres S^{4k-1}, the real projective spaces $\boldsymbol{RP}^{4k-1}$, and spherical space forms in dimension three [18]. Even though Konishi's result mentioned above combined with the earlier work of Wolf [23] on the classification of homogeneous quaternionic Kähler manifolds of positive scalar curvature gives many new homogeneous examples, no further work on 3-Sasakian manifolds seems to have been done until very recently [2, 7].

The purpose of this note is to announce some of our recent results about the geometry of Sasakian 3-structures. Full details and proofs of the results stated below can be found in [2, 3, 4, 5].

Definition A. Let $(\mathcal{S}, g)$ be a Riemannian manifold and let ∇ denote the Levi-Civita connection of g. Then $(\mathcal{S}, g)$ has a Sasakian structure if there exists a Killing vector field ξ of unit length on $\mathcal{S}$ so that the tensor field Φ of type (1,1), defined by

(i) $$\Phi = \nabla\xi$$

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