76. *Resonance in the Cauchy Problem of a Parabolic Equation*

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1. Introduction. Let q be a natural number, and we consider the Cauchy problem of the following strongly parabolic equation of $2q$-th order:

$$\frac{\partial u}{\partial t} = ((-1)^{q-1} + b(t, x)) \frac{\partial^{2q} u}{\partial x^{2q}} \quad t > 0, \quad x \in \mathcal{R}^1, \tag{1}$$

$$u(0, x) = u_0(x), \quad x \in \mathcal{R}^1, \tag{2}$$

where an initial data u_0 and a coefficient b satisfy Assumption 1 (this and the other terminology are defined at §2). In [6], it is proved;

Proposition. *Let Assumption* 1 *hold, then there exists a unique wide sense solution* u *of* (1) *with* (2). *In addition there is a constant* c_∞ *such that*

$$\lim_{t\to\infty} \| u(t,\cdot) - c_\infty \|_0 = 0. \tag{3}$$

Thus in the present note, we announce that c_∞ can be calculated from b and u_0, and its value changes drastically whether u_0 *resonates* with b or not.

On c_∞, only a few results have been known. If one of the following (a) and (b) hold:

(a) $q = 1$, b is real valued and independent of t, and for a constant $\overline{u_0}$,

$$u_0 - \overline{u_0} \in \mathcal{L}_1(\mathcal{R}^1),$$

(b) b is independent of x and there is a constant $\overline{u_0}$ such that

$$\overline{u_0} = \lim_{L\to+\infty} \frac{1}{L} \int_0^L u_0(x)\, dx = \lim_{L\to+\infty} \frac{1}{L} \int_{-L0}^0 u_0(x)\, dx,$$

then it is known in [3, 4, etc.] and [1] that

$$c_\infty = \overline{u_0}. \tag{4}$$

But (4) does not make clear delicate relation between c_∞ and b, because the both conditions above prevent that u_0 *resonates* with b. In this sense, (4) is very different from our result.

Our method to calculate c_∞ is based on an extended Girsanov type formula. The usual Girsanov formula is well known in the theory of probability. It works when first order terms are added to a second order parabolic equation. Besides it, we introduced the extended Girsanov type formula in [5], which works when same order terms are added to a $2q$-th order parabolic equation. By this formula, the wide sense solution u of (1) with (2) is represented in a series, which enables us to calculate c_∞.

2. Notations. Let $\lambda \geq 0$, and let $\mathcal{M}^\lambda(\mathcal{R}^1)$ be a set of all complex valued measures $\mu(d\xi)$ such that

$$\|\mu\|_\lambda \equiv \int_{\mathcal{R}^1} (1 + |\xi|)^\lambda |\mu|(d\xi) < \infty$$

where $|\mu|$ denotes total variation of μ. As well known, $\mathcal{M}^\lambda(\mathcal{R}^1)$ is a Banach