75. *Meromorphic Solutions of Some Second Order Differential Equations*

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1. Introduction. In this note, we investigate the relation between meromorphic solutions of a Riccati equation

(1.1) $$u' + u^2 + A(z) = 0$$

and meromorphic solutions of some second order differential equation

(1.2) $$\varphi'' + 3\varphi'\varphi + \varphi^3 + 4A(z)\varphi + 2A'(z) = 0,$$

where $A(z)$ is a meromorphic function.

For any solutions $u_1(z)$, $u_2(z)$ of (1.1), $\varphi(z) := u_1(z) + u_2(z)$ satisfies the equation (1.2). In fact, denoting by $\Phi(z, \varphi)$ the left-hand side of (1.2), we have

(1.3) $$\Phi(z, \varphi) = 3u_1U_1(z, u_2) + 3u_2U_1(z, u_1) + U_2(z, u_1) + U_2(z, u_2),$$

where $U_1(z, u) = u' + u^2 + A(z)$, $U_2(z, u) = u'' + 3u'u + u^3 + A(z)u + A'(z) = \dfrac{dU_1(z, u(z))}{dz} + uU_1(z, u)$.

It is easy to see that if $u(z)$ satisfies the equation (1.1), then $U_j(z, u(z)) = 0$, $j = 1, 2$. This means that sum $\varphi(z)$ of solutions $u_1(z)$, $u_2(z)$ of the equation (1.1) is a solution of the equation (1.2). Conversely, we get the following theorems:

Theorem 1.1. *Suppose that $A(z)$ is an entire function. Then the equation* (1.2) *admits a meromorphic solution $\varphi(z)$. Moreover, for any meromorphic solution $\varphi(z)$ of* (1.2), *there exist meromorphic solutions $u_1(z)$, $u_2(z)$ of the Riccati equation* (1.1) *such that $\varphi(z) = u_1(z) + u_2(z)$.*

In this note, we use standard notations in the Nevanlinna theory (see, e.g., [3], [6], [7]). Let $f(z)$ be a meromorphic function. As usual, $m(r, f)$, $N(r, f)$, and $T(r, f)$ denote the proximity function, the counting function, and the characteristic function of $f(z)$, respectively. A function $\varphi(r)$, $0 \leqq r < \infty$, is said to be $S(r, f)$ if there is a set $E \subset \mathbf{R}^+$ of finite linear measure such that $\varphi(r) = o(T(r, f))$ as $r \to \infty$ with $r \notin E$. We say that meromorphic solutions $u(z)$ and $\varphi(z)$ are admissible solutions (1.1) and (1.2), if $T(r, A) = S(r, u)$ and $T(r, A) = S(r, \varphi)$, respectively. For some property P, we denote by $n_P(r, c; f)$ the number of c-points in $|z| \leqq r$ that admit the property P. The integrated counting function $N_P(r, c; f)$ is defined in the usual fashion. Suppose $N(r, c; f) \neq S(r, f)$ for a $c \in \mathbf{C} \cup \{\infty\}$. If $N(r, c; f) - N_P(r, c; f) = S(r, f)$, then we say that *almost all* c-points admit the property P.

Theorem 1.2. *Suppose that the equations* (1.1) *and* (1.2) *possess an admissible solution $u_1(z)$ and a meromorphic solution $\varphi(z)$, respectively. If*