74. Some Remarks on the Class of Riemann Surfaces with (W)-property

By Naondô JIN

Department of Mathematics, Gakushuin University

(Communicated by Kiyosi ITÔ, M. J. A., Oct. 12, 1993)

Introduction. In the classification theory we know that some classes of Riemann surfaces are characterized in terms of the subspaces of real square integrable harmonic differentials. For example, $\Gamma_{he}(R) \cap {}^*\Gamma_{he}(R) = \{0\}$ (resp. $\Gamma_{he}(R) \cap {}^*\Gamma_{hse}(R) = \{0\}$) if and only if $R \in O_{AD}$(resp. $R \in O_{KD}$). (See §1 for notations.) In the papers [3,6] M. Watanabe (neeé Mori) introduced the following condition, which we call here (W)-property,

$$\Gamma_{he}(R) \cap {}^*\Gamma_{hse}(R) \subset {}^*\Gamma_{he}(R)$$

or equivalently

$$\Gamma_{ho}(R) \cap {}^*\Gamma_{ho}(R) = \Gamma_{hse}(R) \cap {}^*\Gamma_{ho}(R)$$

for a Riemann surface R. She obtained other equivalent conditions and interesting conseqences.

In the paper [1] we have given a new characterization of (W)-property in terms of specific period reproducing differentials.

In the present paper we shall consider the class of Riemann surfaces with (W)-property, which we denote by P_W, in the context of the classification theory.

1. Preliminaries. For the sake of convenience we recall some definitions. Let $\Gamma_h(R)$ be the Hilbert space of real square integrable harmonic differentials on a Riemann surface R, where the inner product is given by

$$(\omega_1, \omega_2) = (\omega_1, \omega_2)_R = \int\int_R \omega_1 \wedge {}^*\omega_2,$$

${}^*\omega_2$ being the conjugate differential of ω_2. Let $\Gamma_{he}(R)$ (resp. $\Gamma_{hse}(R)$) be the subspace of $\Gamma_h(R)$ whose elements ω are exact (resp. semiexact) on R, that is

$$\int_\gamma \omega = 0 \text{ for every (resp. every dividing) 1-cycle } \gamma \text{ on } R.$$

Given a closed subspace Γ_y of Γ_h, the orthogonal complement of Γ_y in Γ_h is denoted by $\Gamma_y^\perp$. For the spaces $\Gamma_{ho} = ({}^*\Gamma_{he})^\perp$ and $\Gamma_{hm} = ({}^*\Gamma_{hse})^\perp$, the following inclusion relations hold

$$\Gamma_h \supset \Gamma_{hse} \supset \Gamma_{he} \supset \Gamma_{hm}\,;\, \Gamma_{hse} \supset \Gamma_{ho} \supset \Gamma_{hm}.$$

The Γ_{hm} is known as the space of harmonic measure differentials. For a given 1-cycle c on R and a closed subspace Γ_y of Γ_h there exists uniquely the period reproducing differential $\sigma_y(c)$ in Γ_y such that

$$\int_c \omega = (\omega, \sigma_y(c))_R \text{ for every } \omega \in \Gamma_y.$$

We are interested in $\sigma_{hse}(c)$ and $\sigma_{ho}(c)$.