73. *Nonlinear Perron-Frobenius Problem for Order-preserving Mappings. II. —Applications*

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Abstract: In this paper, we apply the results in part I of this paper to boundary value problems for a class of partial differential equations. First, we generalize the Fujita lemma, which is concerned with the properties of solutions of the equation $\Delta u + f(u) = 0$ with strictly convex function f, to the case where f is a convex function. The second example is a bifurcation problem for the semilinear elliptic equation of the form $\Delta u + \lambda f(u) = 0$ under the Dirichlet boundary conditions. We discuss properties of a bifurcation branch of solutions. The third example is a nonlinear (but positively homogeneous) eigenvalue problem.

Key words: Perron-Frobenius; order-preserving; indecomposable; bifurcation; generalized Fujita lemma.

1. Introduction. In part I of the present series of papers, we have extended the Perron-Frobenius theorem to nonlinear mappings on an infinite dimensional space. We have studied the properties of eigenvalues and the corresponding eigenvectors.

In this paper we apply the results in part I to a class of partial differential equations.

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2. Generalized Fujita lemma. In what follows, the numbered 'theorems' and 'remarks', as well as the hypotheses A1, A2, A3,$\cdots$, refer to those presented in part I.

Example 1. Let $\Omega \subset \boldsymbol{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$. We consider the Dirichlet boundary value problem:

$$(1.1) \qquad \begin{cases} \Delta u + f(x, u) = 0 & \text{in } \Omega, \\ u = \varphi & \text{on } \partial\Omega, \end{cases}$$

where φ is a continuous function on $\partial\Omega$. Here $f(x, u) : \bar{\Omega} \times \boldsymbol{R} \rightarrow \boldsymbol{R}$ is locally Hölder continuous in x, u, and locally uniformly Lipschitz continuous in u, that is, for any bounded closed interval $[a, b] \subset \boldsymbol{R}$, there exists some constant $C > 0$ such that

$$\sup_{x \in \Omega} \sup_{\substack{u,v \in [a,b] \\ u \neq v}} \frac{|f(x, u) - f(x, v)|}{|u - v|} \leq C.$$

Hereafter we consider only classical solutions. (It is easily shown that