72. *Nonlinear Perron-Frobenius Problem for Order-preserving Mappings. I*

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(Communicated by Kiyosi ITÔ, M. J. A., Oct. 12, 1993)

Abstract: We consider the eigenvalue problem of an order-preserving mapping defined on a positive cone of an ordered Banach space. Among other things, we prove the existence and, in some cases, the uniqueness of the positive eigenvalue. We also discuss other properties of eigenvalues and eigenvectors. The notion of indecomposability for nonlinear mappings that we introduce in an infinite dimensional setting will play a key role in our argument. We apply the results in this paper to boundary value problems for a class of partial differential equations in part II.

Key words: Perron-Frobenius; order-preserving; indecomposable; maximal eigenvalue; positive eigenvector.

1. Introduction. The Perron-Frobenius theorem, which is concerned with the properties of eigenvalues and eigenvectors of square matrices whose components are nonnegative, has been extended and applied in various ways. It has been generalized to positive linear operators on a Banach space in [1], [3], [6], [12]. From the point of view of applications to mathematical economics, extensions of the theory to nonlinear mappings have also been obtained in [4], [5], [9], [10], [11]. They are, however, concerned only with problems in a finite dimensional Euclidean space.

In this paper, we extend these results to nonlinear mappings on an infinite dimensional space. In doing so, we introduce the notion of indecomposability for a nonlinear mapping on an infinite dimensional space. This notion is an infinite dimensional extension of that for a mapping on an n-dimensional Euclidean space defined in [4; Appendix], and is also a nonlinear extension of that for a linear operator found in [2]. We then consider the eigenvalue problem of an order-preserving mapping T defined on a positive cone E_+ of an ordered Banach space E. We define the operator norm of a positively homogeneous mapping T and denote $\lim_{n\to\infty} \| T^n \|^{1/n}$ by $r(T)$, as in the case of linear operator. The quantity $r(T)$ plays an important role in establishing the existence of positive eigenvalues. This places our problem in marked contrast with the case of finite dimensional spaces, in which the estimation of $r(T)$ is of little importance since the existence of positive eigenvalues is obtained by rather a straightforward application of Brower fixed-point theorem.

As the space is limited, we omit the proof of our theorems. See the forthcoming paper [7] for details.