# 61. *A Note on Jacobi Sums. III*

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This is a continuation of [1] which will be referred to as (II). In this paper, we shall reprove Theorem 2 of (II)[1)] in a setting which suggests us a direction in further studies inspired by Stickelberger's theorem. We follow, in general, notation and conventions of (II). This paper is logically independent of (II).

**§1. Quotient space $H(\mathfrak{P}^{\omega})$.** Let $K/k$ be a finite Galois extension of number fields $K, k$ of finite degree over $\boldsymbol{Q}$ with the Galois group $G = G(K/k)$. Let $\Pi$ be the set of prime ideals $\mathfrak{P}$ of $K$ unramified for $K/k$. We shall call a map $\varphi : \Pi \rightarrow K^{\times}$ a function of type (S) if it satisfies the following conditions:

(S.1) $\varphi(\mathfrak{P}^s) = \varphi(\mathfrak{P})^s$ for all $s \in G$,

(S.2) there is an $\omega_\varphi \in \boldsymbol{Z}[G]$ such that $(\varphi(\mathfrak{P})) = \mathfrak{P}^{\omega_\varphi}$ for all $\mathfrak{P} \in \Pi$.

Using a prime $\mathfrak{p}$ of $k$ which splits completely in $K$, one sees that $\omega_\varphi$ is well-defined by $\varphi$ and that $\omega_\varphi$ belongs to the center $\boldsymbol{Z}[G]_0$ of $\boldsymbol{Z}[G]$. If we denote by $\Phi$ the set of all maps $\varphi$ of type (S), then $\Phi$ becomes a multiplicative group in an obvious way and the map $\varphi \rightarrow \omega_\varphi$ becomes a homomorphism of $\Phi$ into the additive group of $\boldsymbol{Z}[G]_0$ whose kernel consists of all maps $\varphi : \Pi \rightarrow \mathfrak{o}_K^{\times}$, the group of units of $\mathfrak{o}_K$.

As in (II), for $\varphi \in \Phi$, $\omega \in \boldsymbol{Z}[G]$, we put

$$
(1.1)\qquad
\begin{aligned}
G(\varphi(\mathfrak{P})) &= \{s \in G\,;\, \varphi(\mathfrak{P})^s = \varphi(\mathfrak{P})\},\\
G^*(\varphi(\mathfrak{P})) &= \{s \in G\,;\, (\varphi(\mathfrak{P}))^s = (\varphi(\mathfrak{P}))\},\\
G(\mathfrak{P}^{\omega}) &= \{s \in G\,;\, (\mathfrak{P}^{\omega})^s = \mathfrak{P}^{\omega}\}.
\end{aligned}
$$

Note that we use the convention $\mathfrak{P}^{st} = (\mathfrak{P}^t)^s$, $s, t \in G$. Since $\omega_\varphi \in \boldsymbol{Z}[G]_0$ we have, by (S.2),

$$(1.2)\qquad G(\mathfrak{P}^{\omega_\varphi}) = G^*(\varphi(\mathfrak{P})) \supset G(\varphi(\mathfrak{P})) \supset G(\mathfrak{P})$$

where $G(\mathfrak{P})$ means the decomposition group of $\mathfrak{P}$, i.e., $G(\mathfrak{P}) = G(\mathfrak{P}^1)$, $1 \in \boldsymbol{Z}[G]$. For an $\omega \in \boldsymbol{Z}[G]_0$, we shall put

$$(1.3)\qquad H(\mathfrak{P}^{\omega}) = G(\mathfrak{P}^{\omega}) / G(\mathfrak{P}).$$

Write an $\omega \in \boldsymbol{Z}[G]_0$ as

$$(1.4)\qquad \omega = \sum_{t \in G} a(t)t.$$

Since $a = a(t)$ is a class function on $G$, its Fourier expansion makes sense:

$$(1.5)\qquad a = \sum_{\chi \in Irr(G)} a_\chi \chi$$

where $Irr(G)$ denotes the set of $\boldsymbol{C}$-irreducible characters of $G$. The Fourier coefficients are

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1) As for the statement, see the last line of this paper before Acknowledgement.