60. *Value Groups of Henselian Valuations*

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(Communicated by Shokichi IYANAGA, M. J. A., Sept. 13, 1993)

0. Introduction. Let us begin with Neukirch's formulation of general class field theory ([1], [2]). Let G be a pro-finite group and let (G_K) be the closed subgroups of G indexed by "fields" K. Take the "ground field" k such that $G = G_k$. For fields L and K, L is called an extension of K denoted by L/K, if G_K contains G_L and the group index $[G_K : G_L]$ is called the extension degree of L/K denoted by $[L:K]$. Further, if G_L is a normal subgroup of G_K, L is a Galois extension of K with the Galois group $G(L/K) = G_K/G_L$.

For fields K_1 and K_2, the composite field $K_1 K_2$ is defined to be a field such that $G_{K_1K_2} = G_{K_1} \cap G_{K_2}$, and the intersection $K_1 \cap K_2$ is defined to be a field such that $G_{K_1 \cap K_2}$ is the closed subgroup of G generated topologically by G_{K_1} and G_{K_2}.

Let $\hat{\boldsymbol{Z}}$ be the completion of the module $\boldsymbol{Z}$ of rational integers with respect to the finite-index-subgroup-topology. Take a surjective continuous homomorphism deg: $G_k \to \hat{\boldsymbol{Z}}$ and let $\tilde{k}$ be a field such that $G_{\tilde{k}}$ is the kernel of deg. For a finite extension K of k, put $\tilde{K} = K\tilde{k}$ and $f_K = [K \cap \tilde{k} : k]$.

Now suppose that a multiplicative G-module A is given. For a field K let A_K be the submodule of A of elements fixed by G_K. And for a finite extension L of K, we have a homomorphism $N_{L|K} : A_L \ni a \to \Pi_{\sigma \in G_K/G_L} a^{\sigma} \in A_K$.

In [2], Neukirch defined a *Henselian valuation* with respect to deg to be a homomorphism $v : A_k \to \hat{\boldsymbol{Z}}$ satisfying the following two conditions;

(i) the image $Z = v(A_k)$ contains $\boldsymbol{Z}$ and $\boldsymbol{Z}/n\boldsymbol{Z} \simeq Z/nZ$ for any positive integer n,

(ii) $v(N_{K|k} A_K) = f_K \cdot Z$ for any finite extension K of k.

In this paper, any family $(G, A, \deg, v)$ as above will be called an *admissible situation* over k.

We shall study here the structure of the value group Z of a Henselian valuation v and show that if for any finite subextension L/K of $\tilde{K}/K$ the class field axiom

$$^{\#}H^i(G(L/K), A_L) = \begin{cases} [L:K] & \text{if } i = 0 \\ 1 & \text{if } i = -1 \end{cases}$$

holds, then a Henselian valuation v is essentially determined by G, A and deg.

Neukirch has shown that an admissible situation $(G, A, \deg, v)$ gives a "class field theory", if the class field axiom holds for any finite cyclic extension L/K. Thus our result will show that a Henselian valuation v is essentially unique in Neukirch's class field theory.

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