59. *The Restriction of $A_q(\lambda)$ to Reductive Subgroups*

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1. Discrete decomposability with respect to symmetric pairs. Let G be a real reductive linear Lie group and $\hat{G}$ the unitary dual of G. Suppose G' is a reductive subgroup of G. The representation $\pi \in \hat{G}$ is called *G'-admissible* if the restriction $\pi_{|G'}$ splits into a discrete sum of irreducible representations of G' with finite multiplicity. It may well happen that the restriction $\pi_{|G'}$ contains continuous spectrum (even worse, with infinite multiplicity) which is sometimes difficult to analyse. Thus, the notion of admissibility is emphasized here to single out a very nice pair (π, G') for the study of the restriction $\pi_{|G'}$. Here are famous examples where $\pi \in \hat{G}$ is G'-admissible.

(1.1)(a) If G' is a maximal compact subgroup of G, then any $\pi \in \hat{G}$ is G'-admissible (Harish-Chandra). An explicit decompostition formula is known as a *generalized Blattner formula* if $\pi = A_{\mathfrak{q}}(\lambda)$ (attached to elliptic orbits in the sense of orbit method; see [2], [9] Theorem 6.3.12).

(1.1)(b) A restriction formula of a holomorphic discrete series G' is found with respect to some reductive subgroups G' (eg. [7], [4]). Also the restriction of the Segal-Shale-Weil representation π with respect to dual reductive pair with one factor compact is intensively studied (Howe's correspondence).

We remark that G' is compact in the case (1.1)(a), while $\pi \in \hat{G}$ is a highest weight module in (1.1)(b). On the other hand, in some special settings, explicit restriction formulas have been found where $\pi \in \hat{G}$ does not belong to unitary highest weight modules but is G'-admissible for noncompact $G' \subset G$, such as $(G, G') \simeq (SO(4,2), SO(4,1))$ and π is non-holomorphic discrete series ([5] Example 3.4.2), $(G, G') = (SO(4,3), G_2(\boldsymbol{R}))$ and π is in some family of derived functor modules (Kobayashi-Uzawa, 1989 at Math. Soc. Japan), and a recent work of Howe and Tan [3]. See also an explicit formula of the discrete part of $\pi_{|G'}$ for $(G, G') \simeq (SO(3,2), SO(2,2))$ and π non-holomorphic discrete series in [1] in the non-admissible case. In this section we find a more general but still good framework to study the restriction $\pi_{|G'}$.

Let θ be a Cartan involution of G. Write $\mathfrak{g}_0$ for the Lie algebra of G, $\mathfrak{g} = \mathfrak{g}_0 \otimes \boldsymbol{C}$ for its complexification, $K = G^{\theta}$ for the fixed point group of θ, and $\mathfrak{g}_0 = \mathfrak{k}_0 + \mathfrak{p}_0$ for the corresponding Cartan decomposition. Take a fundamental Cartan subalgebra $\mathfrak{h}_0^c (\subset \mathfrak{g}_0)$. Then $\mathfrak{t}_0^c := \mathfrak{h}_0^c \cap \mathfrak{k}_0$ is a Cartan subalgebra of $\mathfrak{k}_0$. A θ-stable parabolic subalgebra $\mathfrak{q} \equiv \mathfrak{q}(\lambda) = \mathfrak{l}(\lambda) + \mathfrak{u}(\lambda) \subset \mathfrak{g}$ and a Levi part $L(\lambda) \subset G$ are given by an elliptic element $\lambda \in \sqrt{-1}(\mathfrak{t}_0^c)^*$ (see [9] Definition 5.2.1). Let $\mathcal{R}_{\mathfrak{q}}^j \equiv (\mathcal{R}_{\mathfrak{q}}^{\mathfrak{g}})^j$ $(j \in \boldsymbol{N})$ be the Zuckerman's derived functor from the category of metaplectic $(\mathfrak{l}, (L \cap K)^{\sim})$-modules to that of $(\mathfrak{g}, K)$-modules. In this paper, we follow the normalization in [10] Definition