58. *Normal Band Compositions of Semigroups**)

By Miroslav ĆIRIĆ and Stojan BOGDANOVIĆ

University of Niš, Yugoslavia

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Abstract: In this paper we give a construction of bands of arbitrary semigroups and we apply this result to study of normal bands of semigroups, and especially for normal bands of monoids. We generalize some well-known results concerning normal bands of monoids and groups.

In this paper we consider band compositions in the general case. Using a general construction for a semilattice of semigroups, we give a construction for a band of arbitrary semigroups. This construction is a very simple consequence of Theorem A, but we give some important applications of this construction: We give a description of normal bands of arbitrary semigroups, especially of normal bands of monoids, and as consequences we obtain some well-known results concerning normal bands of monoids and groups. Note that in our considerations, the conditions (5) and (6) in Theorem A have the important role.

Throughout this paper, $S = (B\,;S_i)$ means that a semigroup S is a band B of semigroups S_i, $i \in B$. Let $S = (B\,;S_i)$, where each S_i is a monoid with the identity e_i, S is a *systematic band* B of S_i, $i \in B$, if $ij = j \Rightarrow e_ie_j = e_j$ and $ji = j \Rightarrow e_je_i = e_j$ (M. Yamada [14]). S is a *proper band* of S_i if $\{e_i \mid i \in B\}$ is a subsemigroup of S (B.M. Schein [11]). Let S be an ideal of a semigroup D. A congruence σ on D is an S-*congruence* on D if its restriction on S is the equality relation on S. An ideal extension D of a semigroup S is a *dense extension* of S if the equality relation is the unique S-congruence on D.

Theorem A [9]. *Let Y be a semilattice. For each $\alpha \in Y$ we associate a semigroup S_α and an extension D_α of S_α such that $D_\alpha \cap D_\beta = \emptyset$ if $\alpha \neq \beta$. For every pair $\alpha, \beta \in Y$ such that $\alpha \geq \beta$ let $\phi_{\alpha,\beta} : S_\alpha \rightarrow D_\beta$ be a mapping satisfying*:

(1) *$\phi_{\alpha,\alpha}$ is the identity mapping on S_α*;

(2) $(S_\alpha\phi_{\alpha,\alpha\beta})(S_\beta\phi_{\beta,\alpha\beta}) \subseteq S_{\alpha\beta}$;

(3) $[(a\phi_{\alpha,\alpha\beta})(b\phi_{\beta,\alpha\beta})]\phi_{\alpha\beta,\gamma} = (a\phi_{\alpha,\gamma})(b\phi_{\beta,\gamma})$,

for all $\alpha, \beta, \gamma \in Y$ such that $\alpha\beta > \gamma$ and all $a \in S_\alpha$, $b \in S_\beta$.

Define a multiplication $*$ on $S = \cup_{\alpha \in Y} S_\alpha$ with:

(4) $$a * b = (a\phi_{\alpha,\alpha\beta})(b\phi_{\beta,\alpha\beta}), \quad (a \in S_\alpha,\ b \in S_\beta).$$

Then S is a semilattice Y of semigroups S_α, in notation $S = (Y\,; S_\alpha, \phi_{\alpha,\beta}, D_\alpha)$.

Conversely, every semigroup S which is a semilattice Y of semigroups S_α can be so constructed. In addition, D_α can be chosen to satisfy:

(5) $D_\alpha = \{b\phi_{\beta,\alpha} \mid \beta \geq \alpha,\ b \in S_\beta,\ \beta \in Y\}$;

(6) *D_α is a dense extension of S_α.*

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