56. *Flat Structure for the Simple Elliptic Singularity of Type $\tilde{E}_6$ and Jacobi Form*

By Ikuo SATAKE

Department of Mathematics, Faculty of Science, Osaka University

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§1. Introduction. In order to construct the inverse mapping of the period mapping for the primitive form for the semi-universal deformation of a simple elliptic singularity, K. Saito introduced in [5] the "flat structure" for the extended affine root system. In section 3, we construct explicitly the flat theta invariants in the case of type E_6 using the Jacobi form introduced by Wirthmüller [7]. Combining the results of Kato [3], Noumi [4] (explicit description of the flat coordinates), this gives an answer to Jacobi's inversion problem (up to linear isomorphism) of this period mapping for a simple elliptic singularity of type $\tilde{E}_6$ (see also [6]). The details will be published elsewhere.

§2. Jacobi form. $\mathfrak{h}_{\boldsymbol{C}}$ is a (complexified) Cartan subalgebra for a fixed simple Lie algebra of rank l. $\mathfrak{h}_{\boldsymbol{C}}^* := Hom_{\boldsymbol{C}}(\mathfrak{h}_{\boldsymbol{C}}, \boldsymbol{C})$. $R^\vee$: the set of coroots. W: Weyl group. $Q(R^\vee)$: the $\boldsymbol{Z}$-span of $R^\vee$. $\langle , \rangle$: the Killing form normalized as $\langle \alpha, \alpha \rangle = 2$ for the highest root α. We identify $\mathfrak{h}_{\boldsymbol{C}}$ with $\mathfrak{h}_{\boldsymbol{C}}^*$ via $\langle , \rangle$. A symmetric tensor

$$\tilde{I}_W := \frac{\partial}{\partial \tau} \otimes \frac{\partial}{\partial t} + \frac{\partial}{\partial t} \otimes \frac{\partial}{\partial \tau} + \sum_{i=1}^{l} \frac{\partial}{\partial z_i} \otimes \frac{\partial}{\partial z_i}, \tag{2.1}$$

is defined on $\boldsymbol{H} \times \mathfrak{h}_{\boldsymbol{C}} \times \boldsymbol{C} \ni (\tau, z, t)$, where $\boldsymbol{H} := \{\tau \in \boldsymbol{C} ; Im\tau > 0\}$, z_i is an orthonormal basis of $\mathfrak{h}_{\boldsymbol{C}}^*$. The symbol $e(x)$ denotes $exp(2\pi\sqrt{-1}x)$.

Definition 2.1. *A Jacobi form of weight k and index m ($k, m \in \boldsymbol{Z}$) is a holomorphic function $\varphi : \boldsymbol{H} \times \mathfrak{h}_{\boldsymbol{C}} \times \boldsymbol{C} \to \boldsymbol{C}$ satisfying*

1) $\varphi(\tau, z + \lambda + \mu\tau, t - \frac{1}{2}\langle \mu, \mu \rangle \tau - \langle \mu, z \rangle) = \varphi(\tau, z, t)$ *for any* λ, $\mu \in Q(R^\vee)$,
2) $\varphi(\tau, w(z), t) = \varphi(\tau, z, t)$ *for any* $w \in W$,
3) $\varphi(\tau, z, t + \alpha) = e(-m\alpha)\varphi(\tau, z, t)$ *for any* $\alpha \in \boldsymbol{C}$,
4) $\varphi\left(\frac{a\tau + b}{c\tau + d}, \frac{z}{c\tau + d}, t + \frac{c\langle z, z \rangle}{2(c\tau + d)}\right) = (c\tau + d)^k \varphi(\tau, z, t)$ *for any* $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \boldsymbol{Z})$,
5) φ *has a Fourier series expansion of the form*
$$e(-mt) \sum_{n \in \boldsymbol{Z}} \phi_n(z) q^n \quad (q = e(\tau))$$
with $\phi_n(z) = 0$ *if* $n < 0$.

The vector space of all Jacobi forms of weight k and index m is denoted by $J_{k,m}$. Put

$$J_{**} = \bigoplus_{k,m \in \boldsymbol{Z}} J_{k,m}, \quad M_* = \bigoplus_{k \in \boldsymbol{Z}} J_{k,0}. \tag{2.2}$$