53. *On the Order of Strongly Starlikeness of Strongly Convex Functions*

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1. Introduction. Let A denote the set of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ that are analytic in $E = \{z : |z| < 1\}$. A function $f(z) \in A$ is called strongly starlike of order β, $0 < \beta \leqq 1$, if $|\arg(zf'(z)/f(z))| < \pi\beta/2$ in E.

Let us denote $\mathrm{STS}(\beta)$ the class of all functions which satisfy the above conditions. On the other hand, a function $f(z) \in A$ is called strongly convex of order β, $0 < \beta \leqq 1$, if $|\arg(1 + zf''(z)/f'(z))| < \pi\beta/2$ in E.

Let us denote $\mathrm{STC}(\beta)$ the class of all functions which satisfy the above conditions.

Mocanu [1, Corollary 1] obtained the following result.

If $f(z) \in A$ and

$$\left|\arg\left(1 + \frac{zf''(z)}{f'(z)}\right)\right| < \frac{\pi\gamma}{2} \text{ in } E,$$

then

$$\left|\arg \frac{zf'(z)}{f(z)}\right| < \frac{\pi\beta}{2}$$

where

$$\mathrm{Tan}\,\frac{\pi\gamma}{2} = \mathrm{Tan}\,\frac{\pi\beta}{2} + \frac{\beta}{(1-\beta)\cos\frac{\pi\beta}{2}}\left(\frac{1-\beta}{1+\beta}\right)^{\frac{1+\beta}{2}}$$

and $0 < \beta < 1$.

In this paper, we will prove the following theorem.

Main theorem. Let $f(z) \in \mathrm{STC}(\alpha(\beta))$. Then we have $f(z) \in \mathrm{STS}(\beta)$, where

$$\alpha(\beta) = \beta + \frac{2}{\pi}\mathrm{Tan}^{-1}\frac{\beta q(\beta)\sin\frac{\pi}{2}(1-\beta)}{p(\beta) + \beta q(\beta)\cos\frac{\pi}{2}(1-\beta)}$$

$$p(\beta) = (1+\beta)^{\frac{1+\beta}{2}} \text{ and } q(\beta) = (1-\beta)^{\frac{\beta-1}{2}}.$$

2. Preliminaries. To prove the main theorem, we need the following lemma.

Lemma. *Let $p(z)$ be analytic in E, $p(0) = 1$, $p(z) \neq 0$ in E and suppose that there exists a point $z_0 \in E$ such that*

$$|\arg p(z)| < \frac{\pi\alpha}{2} \quad \text{for } |z| < |z_0|$$

and

$$|\arg p(z_0)| = \frac{\pi\alpha}{2}$$