34. Pseudo Volume Forms and their Applications to Holomorphic Mappings

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1. A Generalization of Schwarz's lemma. Let M and N be complex manifolds of dimension m and n, respectively and $f : M \to N$ denote a holomorphic mapping. Let θ and ω be the associated 2-forms of hermitian metrics ds_M^2 and ds_N^2 on M and N, respectively. Let Φ be a non-negative (m, m)-form of class C^∞ on M and define a function u by

$$\Phi = u\theta^m. \tag{1}$$

For a function λ on M, define

$$E_\lambda = f^*(Ric\omega^n) - \lambda Ric\Phi. \tag{2}$$

If rank of $f \geq b > 0$ with u_b defined to be

$$\Phi = u_b f^*(\omega^b) \wedge \theta^{m-b} \tag{3}$$

then u can be estimated as follows.

Theorem 1.1. *Let M be a complete Kahler manifold with the Ricci curvature bounded from below and let N be a hermitian manifold with the Ricci curvature bounded from above by a negative constant K_2. Suppose the rank of $f \geq b > 0$. If there exist a constant K_1, a non-negative function λ bounded from above and a non-negative (m, m)-form $\Phi \not\equiv 0$ of class C^∞ such that*

$$\lambda R - Tr(E_\lambda) \geq K_1, \quad sup\, u_b < \infty,$$

where R is the scalar curvature of M, then $K_1 < 0$, and

$$0 < sup\, u \leq \binom{n}{b} \left(\frac{K_1}{bK_2}\right)^b sup\, u_b.$$

As consequences and applications of Theorem 1.1, we exhibit some special and wellknown cases as follows.

Special case 1. Suppose

$$m = n = b, \ \lambda = 1, \ \Phi = f^*(\omega^n).$$

Then $E_1 = 0$, $u_n = 1$. Hence we have $0 < sup\, u \leq \left(\frac{K_1}{nK_2}\right)^n$, which includes the results of Yau [8] and Chern [1].

Special case 2. Suppose

$$m > n = b, \ \lambda = 1, \ \Phi = i_{m-n} f^*(\omega^n) \wedge \varphi \wedge \bar{\varphi}$$

where φ is a holomorphic $(m - n)$-form on M. We can prove

$$E_1 = 0, \quad u_n \leq |\varphi|^2.$$

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