4. *Hasse's Norm Theorem for* K_2

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1. Introduction and definitions. In this note, we shall present a description of Galois groups of the quotient field of 2-dimensional local ring and Hasse principle for K_2 of such fields by using hypercohomology and Lichtenbaum's complex $\boldsymbol{Z}(2)$. This note is an announcement of author's doctor thesis [2].

Unless the contrary is explicitly stated, we shall employ the following notation throughout this paper: For a field K, K_s is a fixed separable closure of K. Let G be a group and M a G-module. We denote M^G by $\Gamma(G, M)$, which is viewed as a functor. The symbol $\boldsymbol{Z}(2)$ stands for Lichtenbaum's complex. For definitions and properties on Lichtenbaum's complex, see [3] and [4]. In this note we shall freely use the standard notations on complexes and objects in derived categories as in [3] and [4].

Let A be a two dimensional complete normal local ring whose residue field F is a finite field, K its quotient field and P the set of all prime ideals of A of height one. For each $\mathfrak{p} \in P$, let $A_\mathfrak{p}$ be the completion of the localization of A at $\mathfrak{p}$, $K_\mathfrak{p}$ its quotient field and $\kappa(\mathfrak{p})$ the residue field of $A_\mathfrak{p}$. Note that by [6], $K_\mathfrak{p}$ is a two dimensional local field and $\kappa(\mathfrak{p})$ is a local field in the usual sense.

We shall construct the complex which represents K_2-idele class group, which is defined in [6]. We define first an auxiliary complex. Under the above notation, let $L_\mathfrak{P}$ be a finite unramified extension of $K_\mathfrak{p}$, where $\mathfrak{P}$ is a prime above $\mathfrak{p}$. Then the complex $Q(L_\mathfrak{P})$ [1] is defined to be the mapping cone of the following morphism of complexes:

$$\tau_{\leq 2}\boldsymbol{R}\Gamma(H_\mathfrak{p}, \boldsymbol{Z}(2)) \to F(\mathfrak{p})^\times[-2],$$

where $H_\mathfrak{p} = \mathrm{Gal}((K_s)_\mathfrak{p}/L_\mathfrak{P})$ and $F(\mathfrak{p})$ is the residue field of $L_\mathfrak{P}$.

We also define K_2-idele complex. Let L be a finite extension of K. The complex $I(L)$ is defined as follows. First we set

$$I^S(L) = \prod_{\mathfrak{p}\in S} \tau_{\leq 2}\boldsymbol{R}\Gamma(H_\mathfrak{p}, \boldsymbol{Z}(2)) \times \prod_{\mathfrak{p}\in P-S} Q(L_\mathfrak{P}),$$

for a finite subset S of P containing all the ramified primes in L/K. Then the $I(L)$ is defined by

$$I(L) = \varinjlim_S I^S(L).$$

The idele complex $\boldsymbol{I}_K$ is defined as

$$\boldsymbol{I}_K = \varinjlim_L I(L),$$

where the limit runs through all finite extensions of K.

Now we can define our K_2-idele class complex. The complex $C(L)$ is