# 27. *A Partial Criterion for Ergodicity of Geodesic Flows on Surfaces with Infinite Area and Negative Curvature*

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**Abstract**: Our problem is under what conditions geodesic flows on surfaces with infinite area and non-constant negative curvature are ergodic. Our result is that ergodicity is preserved under the change of the non-Euclidean metric on any compact set.

**1. Criteria in the case of constant negative curvature.** Let $U = \{(x, y) \in R^2 ; x^2 + y^2 = 1\}$ be the unit circle and $D = \{(x, y) \in R^2 ; x^2 + y^2 < 1\}$ be the interior of $U$. We give on $D$ a non Euclidean metric

$$dc^2 = 4(dx^2 + dy^2) / (1 - x^2 - y^2)^2.$$

We give a discrete subgroup $\Gamma$ of linear fractional transformations which map $D$ to $D$ and $U$ to $U$. Let $M = D/\Gamma$. Then we have a 2-dimensional Riemannian manifold $(M, c)$. We denote by $d_c$ the distance of $M$ introduced from $dc$. We denote by $(\gamma_v^{c,p}(t))$ the geodesic of $(M, c)$ determined by an initial point $p \in M$ and an initial tangent vector $v$. We denote by $S_p^c(M)$ the unit tangent vectors $v$ at $p$. Let $e \in S_p^c(M)$ be the unit tangent vector parallel to the positive $x$-axis. We can identify $v \in S_p^c(M)$ with the angle which $v$ forms to $e$. The angular measure of $A \subset S_p^c(M)$ is denoted by $|A|_c$. E. Hopf classified $(M, c)$ into the following two classes ([3], p. 271, Definition).

**Definition (C).** $(M, c)$ is called of first class, if for some (or equivalently any) fixed $p \in M$,

$$|\{v \in S_p^c(M) ; \lim_{t\to\infty} d_c(\gamma_v^{c,p}(t), p) = \infty\}|_c = 0.$$

$(M, c)$ is called of second class, if it is not of first class.

The following theorem is due to E. Hopf, M. Tsuji, etc. (see [3], p. 273, Hauptsatz 5.2 and p. 280, Hauptsatz 7.2).

**Alternative theorem (C)** ([5], [6]). *The following three conditions are equivalent*: (a) $(M, c)$ *is of first class,* (b) $\Gamma$ *is of divergence type,* (c) *the geodesic flow of* $(M, c)$ *is ergodic.*

*Moreover, the following three conditions are equivalent*: (d) $(M, c)$ *is of second class,* (e) $\Gamma$ *is of convergence type,* (f) *the geodesic flow of* $(M, c)$ *is dissipative.*

**2. Theorem.** Let $f$ be a $C^\infty$-function on $D$ such that inequalities $0 < c_1 \leqq f(x, y) \leqq c_2$ hold on $D$, where $c_1$ and $c_2$ are constants. We give a Riemannian metric $ds$ by

$$ds^2 = f^2(x, y)(dx^2 + dy^2) / (1 - x^2 - y^2)^2$$