21. *The Godbillon-Vey Invariant and the Foliated Cobordism Group*[*)]

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Introduction. In this paper we show the following statement: Let $\mathcal{F}$ be a codimension-1 transversely oriented foliation of a closed oriented 3-manifold M. The Godbillon-Vey invariant of $\mathcal{F}$ is zero if and only if $\mathcal{F}$ is foliated cobordant to a codimension-1 transversely oriented foliation $\mathcal{G}$ of a closed oriented 3-manifold N and there exists a sequence $\mathcal{G}_k$ of null-cobordant codimension-1 foliations of N converging to $\mathcal{G}$.

Two codimension-1 transversely oriented foliations $(M, \mathcal{F})$ and $(N, \mathcal{G})$ of closed oriented n-manifolds are *foliated cobordant* if there exists a codimension-1 transversely oriented foliation $(W, \mathcal{H})$ of a compact oriented $(n+1)$-manifold such that $\partial W=(-M)\cup N$, $\mathcal{H}$ is transverse to ∂W and the restrictions $\mathcal{H}|M$ and $\mathcal{H}|N$ coincide with $\mathcal{F}$ and $\mathcal{G}$, respectively. The foliated cobordism classes form an additive group $\mathcal{F}\Omega_{n,1}$. The foliations $(M, \mathcal{F})$ representing the zero of the foliated cobordism group are those cobordant to the empty set. We say they are *null-cobordant.*

The Godbillon-Vey invariant for a codimension-1 transversely oriented foliation $\mathcal{F}$ was defined as follows ([7]). Let ω be a 1-form defining $\mathcal{F}$. The integrability condition is the existence of 1-form η such that $d\omega=\omega\wedge\eta$. Then the 3-form $\eta\wedge d\eta$ is closed and its cohomology class depends only on the foliation $\mathcal{F}$. If $\mathcal{F}$ is a codimension-1 transversely oriented foliation of a closed oriented 3-manifold M, then the Godbillon-Vey invariant is the integral of this 3-form.

There are two properties which follow easily from the definition ([7]). One is that this invariant depends only on the cobordism class of the foliations. This is an easy consequence of the Stokes theorem. The other is that this invariant varies continuously when we deform the foliation. The reason is that the 1-form η can be taken to be the Lie derivative $L_X\omega$, where X is a vector field such that $\omega(X)=1$. The examples for these continuous variations were given by Thurston ([15]), and hence we have a surjective homomorphism $GV: \mathcal{F}\Omega_{3,1}\to \boldsymbol{R}$. The natural question on the injectivity is still an open question.

We can ask a weaker question. By the property of continuous variation of GV, if a foliation is approximated by null-cobordant foliations, its GV is zero. Moreover, if a foliation is cobordant to such an approximable foliation, then its GV is zero. Now the weaker question is whether the

*) This paper is dedicated to the memory of Itiro Tamura (1926–1991).