# 75. On the Dirichlet Form on a Lusinian State Space

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**1. Introduction.** The Dirichlet forms on locally compact state spaces have been studied by many authors. Recently this theory of Dirichlet forms has been extended to non-locally compact state spaces. Albeverio and Ma [1] gave a necessary and sufficient condition for the Dirichlet form on a metrizable topological state space to be associated with a special standard process. They called this Dirichlet form quasi-regular (cf. [3]). On the other hand, Shigekawa and Taniguchi [12] showed that various results known for locally compact state spaces, such as the Beurling-Deny formula, the uniqueness of the $\alpha$-potentials, are also valid for Lusinian separable metric state spaces. The key lemma in [12] is a uniqueness statement for a measure which charges no set of zero capacity. Its proof needs the Gel'fand compactification (cf. [4], [9]). To use the Gel'fand compactification we must assume that there exists a dense subset consisting of continuous functions in the domain of the Dirichlet form. However, this assumption is not necessary for the existence of the associated process (cf. [1]). In fact Albeverio, Röckner and Ma [3] showed the same results for quasi-Dirichlet form on general state spaces. They also used another type of compactification (cf. [10]).

In this note we shall show for the quasi-regular Dirichlet form the uniqueness statement of a measure charging no set of zero capacity without using any type of compactification.

**2. Preliminary.** Let $X$ be a Lusinian separable metric space and let $\mathcal{B}(X)$ be its topological Borel field. Let $\rho$ be its metric. We fix a probability measure $m$ on $(X, \mathcal{B}(X))$ such that $\mathrm{supp}[m] = X$.

We consider a Dirichlet form $(\mathcal{E}, \mathcal{F})$ on $L^2(X, m)$ (for its definition see e.g. [8]). We set

(2.1) $$\mathcal{E}_1(f, g) \equiv \mathcal{E}(f, g) + (f, g), \quad f, g \in \mathcal{F},$$

where $(\cdot, \cdot)$ denotes the inner product of $L^2(X, m)$.

For an open subset $G$ of $X$ and any subset $A$ of $X$, we define

(2.2) $$\mathrm{Cap}(G) \equiv \inf\{\mathcal{E}_1(u, u)\ ;\ u \in \mathcal{F} \text{ and } u \geq 1\ m\text{-a.e. on } G\},$$

(2.2) $$\mathrm{Cap}(A) \equiv \inf\{\mathrm{Cap}(G)\ ;\ G \text{ is open and } A \subset G\}.$$

Then we can show that this Cap is a Choquet capacity.

A statement depending on $x \in A$ is said to hold "*quasi-everywhere*" or simply "*q.e.*", if it holds on $A$ except for a set of zero capacity with respect to Cap. A function $u : X \to \boldsymbol{R}$ is said to be quasi-continuous if there exists a decreasing sequence $\{G_n\}_{n=1}^{\infty}$ of open sets such that $\mathrm{Cap}(G_n) \downarrow 0$, and $u|_{X \setminus G_n}$ is continuous on each $X \setminus G_n$.

**3. The main theorem.** We assume that the Dirichlet form $(\mathcal{E}, \mathcal{F})$ satis-