73. *Dimension Estimate of the Global Attractor for Resonant Motion of a Spherical Pendulum*

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1. Introduction and result. In [8] Miles derived the following system (SP). It describes the motion of a lightly damped spherical pendulum, which is forced to oscillate horizontally in the neighborhood of resonance:

$$\text{(SP)}\qquad \begin{cases} \dfrac{dp_1}{dt} = -\alpha p_1 - \left(\nu + \dfrac{E}{8}\right) q_1 - \dfrac{3}{4} M p_2, \\ \dfrac{dq_1}{dt} = -\alpha q_1 + \left(\nu + \dfrac{E}{8}\right) p_1 - \dfrac{3}{4} M q_2 + 1, \\ \dfrac{dp_2}{dt} = -\alpha p_2 - \left(\nu + \dfrac{E}{8}\right) q_2 + \dfrac{3}{4} M p_1, \\ \dfrac{dq_2}{dt} = -\alpha q_2 + \left(\nu + \dfrac{E}{8}\right) p_2 + \dfrac{3}{4} M q_1, \end{cases}$$

where $\alpha > 0$ and $\nu \in \boldsymbol{R}$ represent a damping coefficient and a frequency offset, respectively. Here $(p_1(t), q_1(t), p_2(t), q_2(t))$ denotes slowly varying amplitudes of degenerate modes 1 and 2 in a four dimensional phase space, and we have set $E = E(t) := p_1(t)^2 + q_1(t)^2 + p_2(t)^2 + q_2(t)^2$, $M = M(t) := p_1(t)q_2(t) - p_2(t)q_1(t)$.

The aim of this paper is to estimate an upper bound for the dimension of X analytically. Basically we make use of the Kaplan-Yorke formula. This formula connects the upper bound with the Lyapunov exponents. This was conjectured by Kaplan and Yorke [7] and proved by Constantin and Foias [1]. In Eden, Foias and Temam [4], this enables to estimate the dimension of a global attractor for the Lorenz system. (SP) consists of four equations unlike the Lorenz system. We therefore adopt the technique used in Ishimura and Nakamura [6].

Now we state our main result.

Theorem. *Let X be the maximal compact invariant set of* (SP). *Let* $\dim_{\mathscr{H}}$ *denote the Hausdorff dimension. For any $\nu \in \boldsymbol{R}$, we have the following:*

(i) *If* $0 < \alpha^3 \le \frac{1}{3}$, *then*

$$\dim_{\mathscr{H}}(X) \le 3 + \frac{-3\alpha^3 + 1}{\alpha^3 + 1}.$$

(ii) *If* $\frac{1}{3} < \alpha^3 \le \frac{9}{16}$, *then*

$$\dim_{\mathscr{H}}(X) \le 2 + \frac{-16\alpha^3 + 9}{8\alpha^3 + 1}.$$