72. *Nonconvex-valued Differential Inclusions in a Separable Hilbert Space*

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1. Introduction. Let $\mathfrak{B}$ be a real separable Hilbert space which is densely and continuously imbedded in another real separable Hilbert space $\mathfrak{H}$. A correspondence (= multi-valued mapping) $\Gamma : [0, T] \times \mathfrak{B} \twoheadrightarrow \mathfrak{H}$ is assumed to be given. We consider the differential inclusion:

$$\dot{x} \in \Gamma(t, x),\ x(0) = 0.$$

Maruyama [7] examined a differential inclusion of this type in the case $\mathfrak{B} = \mathfrak{H}$ and established the existence of solutions under rather restrictive assumptions. In particular the following two assumptions are not satisfied in many important situations:

(i) the correspondence $x \mapsto\!\!\!\to \Gamma(t, x)$ is upper hemi-continuous with respect to the weak topology for $\mathfrak{B}$ and the strong topology for $\mathfrak{H}$, and

(ii) the correspondence Γ is convex-valued.

The first assumption can be weakened to:

(i′) the correspondence $x \mapsto\!\!\!\to \Gamma(t, x)$ is upper hemi-continuous with respect to the weak topologies for both of $\mathfrak{B}$ and $\mathfrak{H}$,

under the additional assumption that Γ is bounded. (See section 6.)

However it seems quite hard to drop the second assumption without any serious change of the proof. In fact, Maruyama [8] exemplified the importance of assumption (ii) in deducing several properties of differential inclusions including the existence of solutions.

In this paper, we shall show the way leading to the existence without having recourse to assumption (ii). Examples will be shown in section 5.

2. Assumptions. We begin by specifying some assumptions imposed on the correspondence $\Gamma : [0, T] \times \mathfrak{B} \twoheadrightarrow \mathfrak{H}$. Denote by $\mathfrak{B}^w$ (resp. $\mathfrak{H}^w$) the space $\mathfrak{B}$ (resp. $\mathfrak{H}$) endowed with the weak topology.

Assumption 1. *The set $\Gamma(t, x) \subset \mathfrak{H}^w$ is nonempty and weakly compact for all $(t, x) \in [0, T] \times \mathfrak{B}$.*

Assumption 2. *For each fixed $t \in [0, T]$, the correspondence $x \mapsto\!\!\!\to \Gamma(t, x)$ is continuous with respect to the weak topologies for both of $\mathfrak{B}$ and $\mathfrak{H}$; i.e. Γ satisfies both the upper hemi-continuity and the lower hemi-continuity in x.* (For the concept of "continuity" of a correspondence, see Aubin-Frankowska [1] Chap. 1.)

Assumption 3. *For each fixed $x \in \mathfrak{B}$, the correspondence $t \mapsto\!\!\!\to \Gamma(t, x)$ is measurable in the sense that the weak inverse image $\Gamma^{-w}(U) = \{t \in [0, T] : \Gamma(t, x) \cap U \neq \emptyset\}$ is measurable for all open sets U in $\mathfrak{H}^w$ and for each fixed $x \in \mathfrak{B}$.* (For the concept of "measurability" of a correspondence,