71. The Generalized Confluent Hypergeometric Functions[†)]

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Introduction. The purpose of this note is to introduce a class of hypergeometric functions of confluent type defined on the Grassmannian manifold $G_{r,n}$, the moduli space for r-dimensional linear subspace in $\boldsymbol{C}^n$. These functions will be called the generalized confluent hypergeometric functions.

Let r and $n(n > r)$ be positive integers and let $Z_{r,n}$ be the set of $r \times n$ complex matrices of maximal rank. On $Z_{r,n}$ there are natural actions of $GL(r, \boldsymbol{C})$ and of $GL(n, \boldsymbol{C})$ by the left and right matrix multiplications, respectively, and the Grassmannian manifold $G_{r,n}$ is identified with the space $GL(r, \boldsymbol{C})\backslash Z_{r,n}$. Let $\varphi : Z_{r,n} \rightarrow G_{r,n}$ be the natural projection map. In Section 1, we define the system of partial differential equations on $Z_{r,n}$ which will be called the generalized confluent hypergeometric system. This system induces the system on $G_{r,n}$ through the mapping φ (see Section 1).

There is given a partition of n, $\lambda = (\lambda_1, \ldots, \lambda_l)$, i.e. the sequence of positive integers $\lambda_1 \geq \cdots \geq \lambda_l > 0$ satisfying $|\lambda| = \lambda_1 + \cdots + \lambda_l = n$. For a partition λ, we define the maximal commutative subgroup H_λ of $GL(n, \boldsymbol{C})$ (see the definition in Section 1) which acts on $Z_{r,n}$ as a subgroup of $GL(n, \boldsymbol{C})$. Our generalized confluent hypergeometric functions $F(z)$ on $Z_{r,n}$ will be a multi-valued analytic function satisfying the homogeneity property:

$$(1)\qquad \begin{cases} F(zc) = F(z)\chi_\alpha(c) & \text{for } c \in H_\lambda, \\ F(gz) = (\det g)^{-1}F(z) & \text{for } g \in GL(r, \boldsymbol{C}), \end{cases}$$

where χ_α is a character of the universal covering group of H_λ (see Section 1). This property implies that the functions $F(z)$ in $Z_{r,n}$ induces multi-valued functions on the quotient space $X_\lambda := G_{r,n}/H_\lambda$. In the case $\lambda = (1, \ldots, 1)$, the confluent hypergeometric function $F(z)$ coincides with the general hypergeometric function of I. M. Gelfand [1] and in the case $\lambda = (n)$, it coincides with the generalized Airy function due to Gelfand, Retahk and Serganova [3].

1. Generalized confluent hypergeometric functions. The Jordan group $J(m)$ of size m is a commutative subgroup of $GL(m, \boldsymbol{C})$ defined by

$$J(m) := \left\{c = \sum_{i=0}^{m-1} c_i \tau^i\,;\ c_i \in \boldsymbol{C},\ c_0 \neq 0\right\},$$

where

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