67. *Algebraic Tori Admitting Finite Central Coregular Extensions*

By Haruhisa NAKAJIMA*)

Department of Mathematics, Keio University

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Abstract: We determine representations of algebraic complex tori admitting finite central coregular extensions.

Key words: Algebraic torus; coregular representation; affine semigroup ring.

1. Introduction. Let $\rho : G \to GL(V(\rho))$ be a finite dimensional rational representation of a complex reductive algebraic group G over the field $\boldsymbol{C}$ of complex numbers, where $V(\rho)$ denotes the representation space. A representation $\varphi : H \to GL(V(\varphi))$ of an algebraic group H is said to be a *finite extension* of ρ or of (ρ, G), if $V(\varphi) = V(\rho)$ and there is a morphism $\psi : G \to H$ such that $\rho = \varphi \circ \psi$ and the index of the canonical image of G in H is finite. Moreover if (φ, H) is coregular, i.e., if its associated quotient variety $V(\varphi)/H = \mathrm{Spec}(\boldsymbol{C}[\varphi]^H)$ is an affine space, then (φ, H) or H is said to be a *finite coregular extension* of ρ and we also say that ρ admits a finite coregular extension, where $\boldsymbol{C}[\varphi]$ denotes the affine coordinate ring of $V(\varphi)$. A finite extension (φ, H) of ρ is said to be *central*, if H is generated by the union of G and the centralizer $Z_H(G)$ of G in H. According to [7], in 1991, D. Shmel'kin has classified all finite coregular extensions of irreducible representations of connected complex simple algebraic groups. Recently, in [7], D. I. Panyushev has defined finite coregular extensions and showed that the associated quotient varieties of the representations of connected semisimple algebraic groups admitting finite coregular extensions are complete intersections. This implies that D. Shmel'kin's classification is *a priori* related to the author's one in [5] (cf.[7]).

Hereafter G stands for a connected complex algebraic torus. Simplicial torus embeddings are defined in [3]. The purpose of this paper is to show

Theorem 1.1. *(ρ, G) admits a finite central coregular extension if and only if the rational convex polyhedral cone associated with the torus embedding $V(\rho)/G$ is simplicial.*

As an easy consequence of this theorem, we obtain the following criterion: Let $\{Y_1, \ldots, Y_n\}$ be a basis of $V(\rho)$ on which $\rho(G)$ is a diagonal subgroup of $GL_n(\boldsymbol{C})$. Let Γ denote a set consisting of all minimal subsets Λ of $\{1, \ldots, n\}$ such that nonzero weights in each subspace $\sum_{i \in \Lambda} \boldsymbol{C} Y_i$ generate a

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